Chapter-10 Electromagnetic waves & propagation. Source free medium $\to \nabla\cdot E = 0$

$\nabla\cdot D = \rho_v \to \quad \nabla\cdot E = \frac{\rho_v}{\epsilon} \leftarrow \rho_v = 0 \quad \boxed{\nabla\cdot E = 0}$

Lorentz - $\nabla\cdot E = -\mu\sigma\frac{\partial E}{\partial t}$,

$\nabla\cdot B = 0$ —

$\nabla\times E = -\frac{\partial B}{\partial t} = -\mu\frac{\partial H}{\partial t}$

If Source free medium - $\rho_v = 0 \Rightarrow \nabla\cdot E = 0$

$\nabla\times H = J_c + \frac{\partial D}{\partial t} = \sigma E + \frac{\partial E}{\partial t}$ —

$-\nabla^2 E = -\sigma\mu\frac{\partial E}{\partial t} - \mu\epsilon\frac{\partial^2 E}{\partial t^2}$

from -

$\nabla\times E = -\mu\frac{\partial H}{\partial t}$

$\frac{\partial}{\partial t}(E_0 e^{j\omega t}) \to E = E_0 e^{j\omega t} \leftarrow$ time harmonic version.

$-\nabla^2 E = -\sigma\mu j\omega E - \mu\epsilon(j\omega)^2 E$

$\nabla\times(\nabla\times E) = -\mu\frac{\partial(\nabla\times H)}{\partial t}$

$-\nabla^2 E = (-\sigma\mu j\omega + \mu\epsilon\omega^2)E$

$+\nabla^2 E = +\sigma\mu\omega\left(j - \frac{\epsilon}{\sigma}\omega\right)E$

$\nabla(\nabla\cdot E) - \nabla^2 E = -\mu\frac{\partial}{\partial t}\left[\sigma E + \epsilon\frac{\partial E}{\partial t}\right]$

$+\nabla^2 E = \sigma\mu\omega j\left(1 + j\frac{\epsilon}{\sigma}\omega\right)E$

$\cancel{\nabla(\nabla\cdot E)} - \nabla^2 E = -\mu\sigma\frac{\partial E}{\partial t} - \mu\epsilon\frac{\partial^2 E}{\partial t^2}$

$\nabla^2 E = j\omega\mu(\sigma + j\omega\epsilon)E$

$$\nabla^2 E = \underline{j\omega\mu(\sigma + j\omega\epsilon)}\, E$$

γ^2 ← propagation constant/ Propagation vector.

$$\boxed{\nabla^2 E = \gamma^2 E}$$ ← wave equation. ← Helmoltz Equation.

$$\boxed{\nabla^2 H = \gamma^2 H}$$

$$\boxed{\nabla^2 \psi = \gamma^2 \psi}, \quad \psi \in E, H.$$

$$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$$

$$\boxed{\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}$$

$\gamma = \alpha + j\beta$ ← Phase constant.

α: attenuation Constant

Source —— destination

P_i P_o

E_y

$P_o < P_i$

$$\gamma = \alpha + j\beta \Rightarrow \gamma^2 = \alpha^2 - \beta^2 + 2j\alpha\beta \quad \text{—(1)} \Rightarrow \mathrm{Re}(\gamma^2) = \alpha^2 - \beta^2 \quad \text{—(2)}$$

$$|\gamma^2| = \alpha^2 + \beta^2 \quad \text{—(3)}$$

$$\rightarrow \gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\rightarrow \gamma^2 = j\omega\mu(\sigma + j\omega\epsilon) \quad \text{—(4)} \Rightarrow \rightarrow |\gamma^2| = \omega^2\mu^2\sigma^2 + \omega^4\mu^2\epsilon^2$$

$$\gamma^2 = j\omega\mu\sigma - \omega^2\mu\epsilon$$

$$|\gamma^2| = \omega\mu\sqrt{\sigma^2 + \omega^2\epsilon^2} \quad \text{—(5)} = \alpha^2 + \beta^2$$

$$\mathrm{Re}(\gamma^2) = -\omega^2\mu\epsilon = \alpha^2 - \beta^2 \quad \text{—(6)}$$

from (5) & (6)

$$\alpha^2 + \beta^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\epsilon^2}$$

$$\beta^2 - \alpha^2 = \omega^2\mu\epsilon$$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2} - 1\right]} \quad \& \quad \beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}$$

$$\nabla^2 E = \gamma^2 E$$

$$\cancel{\frac{\partial^2 E}{\partial x^2}}^{0} + \cancel{\frac{\partial^2 E}{\partial y^2}}^{0} + \frac{\partial^2 E}{\partial z^2} = \gamma^2 E$$

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \gamma^2 E_x}$$

$$D^2 E = \gamma^2 E$$

$$(D^2 - \gamma^2) E = 0$$

$$D = \pm \gamma$$

$$E = A e^{\gamma z} + B e^{-\gamma z}$$

$$E = E_0^- e^{+j\gamma z} + E_0 e^{-j\gamma z}$$

Suppose - ① wave is travelling in +z - direction

② It contains x-component.

③ wave is a function of space & time.

$$E = f(z, t)$$

$e^{+\gamma z}$ ← wave propagation in -ve z ← direction.

$e^{-\gamma z}$ ← wave propagation in +ve z direction.

$$E = E_0^+ e^{-\gamma z}$$

Time harmonic version -

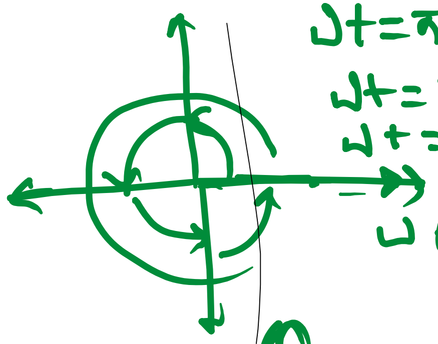
$$E = E_0^+ e^{-\gamma z} e^{j\omega t}$$

$e^{j\omega t}$

$e^{-j\omega t}$

$e^{-j\omega t} = \cos\omega t + j\sin\omega t$

$\omega t = 0$	1	0
$\omega t = \pi/2$	0	1
$\omega t = \pi$	-1	0
$\omega t = 3\pi/2$	0	-1
$\omega t = 2\pi$	1	0



Time harmonic

$E = E_0 e^{-\gamma z} e^{j\omega t}$

$$\boxed{E = E_0 e^{j(\omega t - \gamma z)}}$$

$\rightarrow Re(E) = E_0 \cos(\omega t - \gamma z)$

$\rightarrow Im(E) = E_0 \sin(\omega t - \gamma z)$

α, β

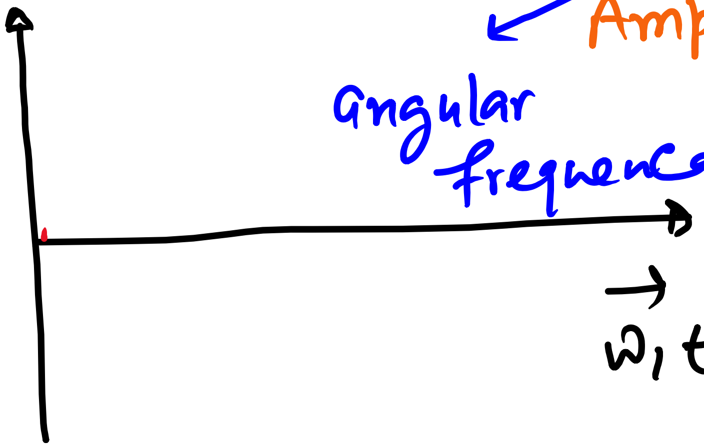
$Im(E) = E_0 \sin(\omega t - \cancel{\gamma} z)$

$E = E_0 e^{-\alpha z} \sin(\omega t - \beta z)$

Amplitude

Phase

angular frequency



$E = E_0 e^{-\gamma z} e^{-j\omega t}$

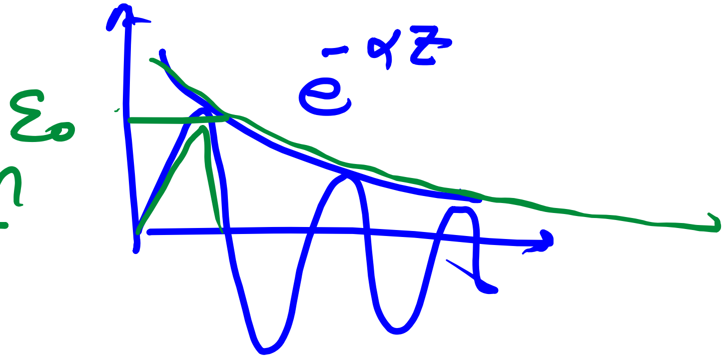
$E = E_0 e^{-\alpha z} e^{j\beta z} e^{-j\omega t}$

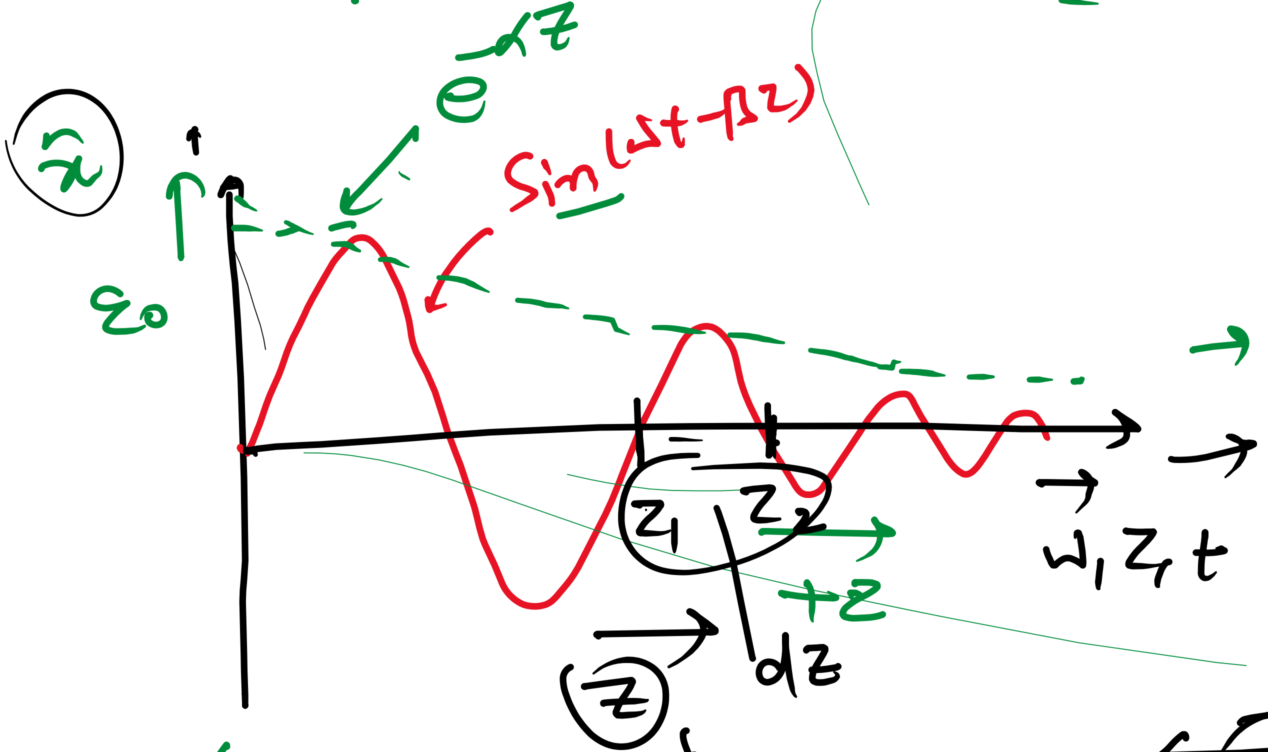
$E = E_0 e^{-\gamma z} e^{-j(\omega t - \beta z)}$

$Im(E) = E_0 e^{-\alpha z} \sin(\omega t - \beta z)\,\hat{x}$

$\rightarrow E = E_0 e^{-\alpha z} \sin(\omega t - \beta z)\,\hat{x}$

+ve z direction

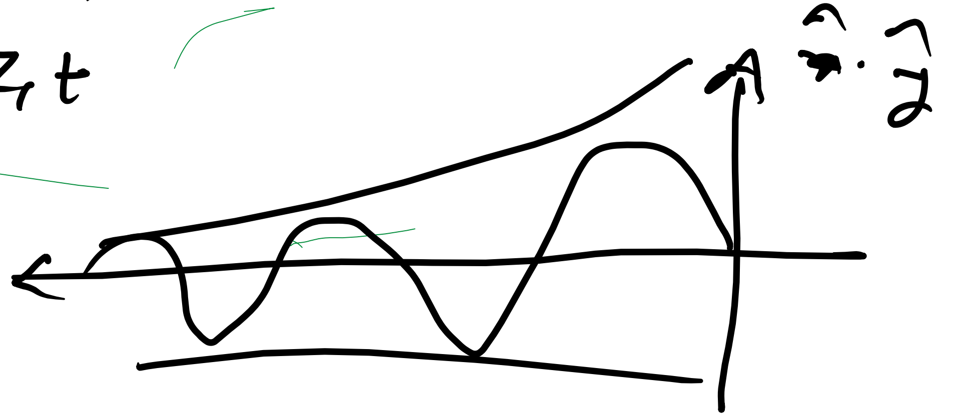




$\rightarrow \mathcal{E}_x = \mathcal{E}_0 e^{-\alpha z} \sin(\omega t - \beta z)\,\hat{x}$

$\rightarrow \mathcal{E} = \mathcal{E}_0 e^{-\alpha z} \sin(\omega t + \beta z)\,\hat{x}$

$\hat{y}$



$$\vec{E} = \varepsilon_0 \, e^{-\alpha x} \sin(\omega t + \beta x)\, \hat{y}$$

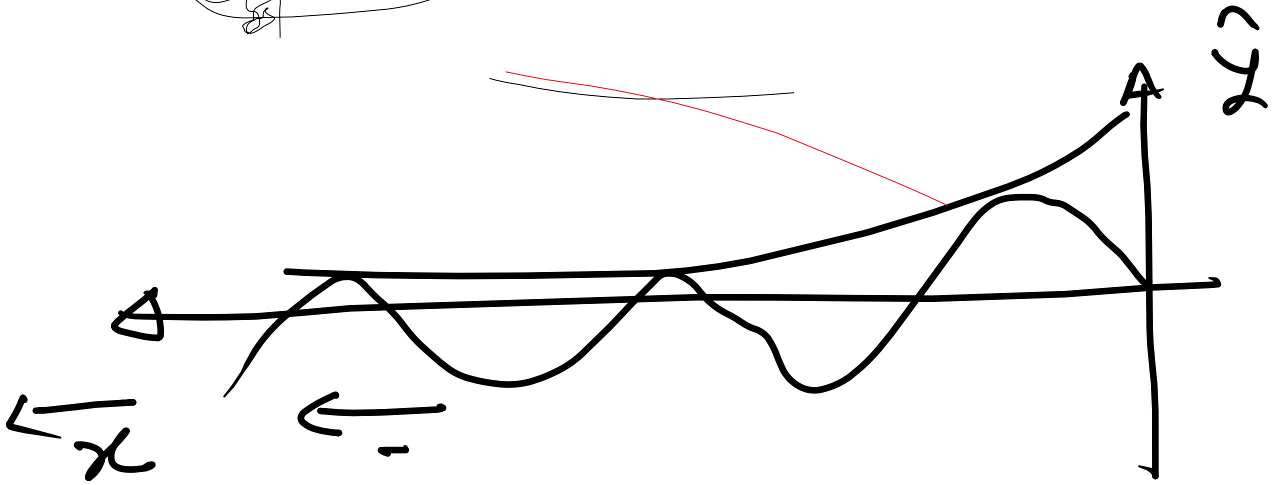
Phase

$-$ve x direction

$$\boxed{v_p = c}$$

free space

$$v_p = \frac{\cancel{2\pi f}\, c}{\cancel{2\pi f}}$$



$$\boxed{\frac{\omega}{\beta} = v_p}$$

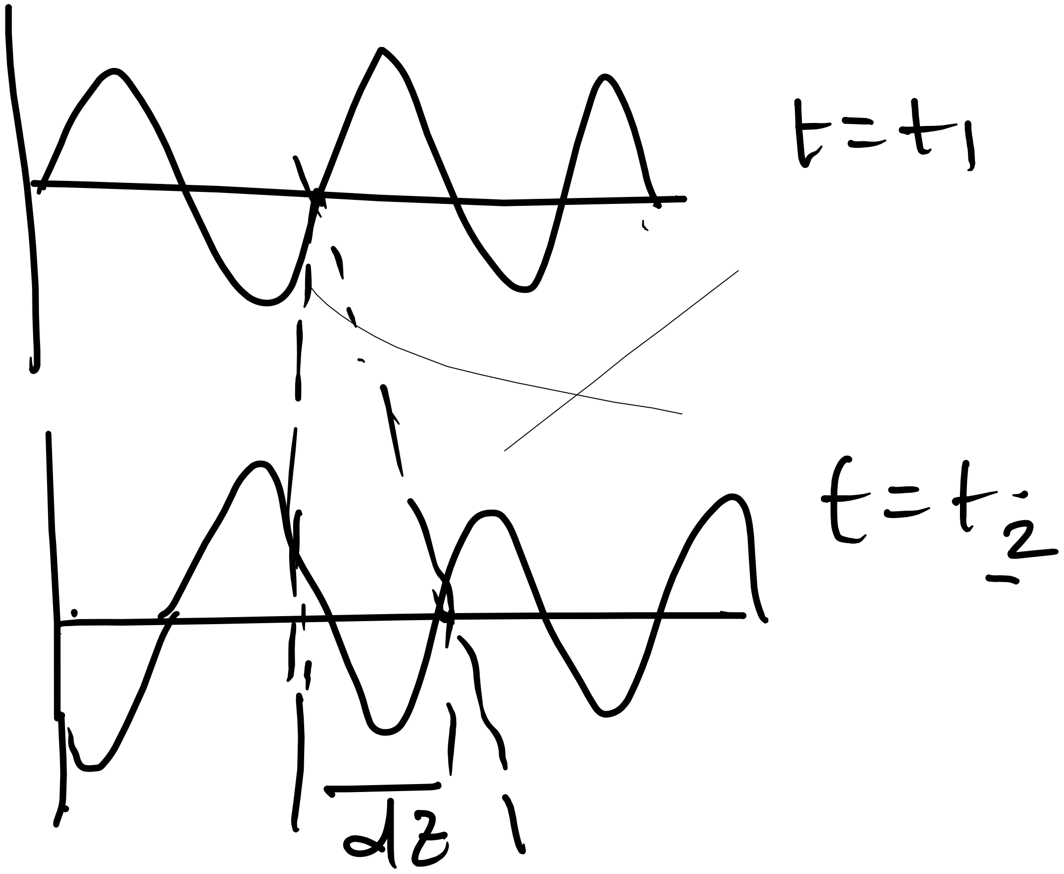
Phase velocity

$$\beta = \frac{2\pi}{\lambda}$$

Phase: $\Rightarrow \omega t - \beta z = \text{Constant}$

$$\lambda = \frac{c}{f}, \quad \omega = 2\pi f$$

$$\omega \frac{dt}{dt} - \beta \frac{dz}{dt} = 0 \Rightarrow \frac{\omega}{\beta} = \frac{dz}{dt}$$



$$\lambda = \frac{\lambda_0}{\sqrt{\mu\epsilon}}$$

$$\boxed{v_p = \frac{c}{\sqrt{\mu\epsilon}}}$$

not free space

$$\left.\begin{aligned} &\rightarrow \nabla^2 E = \gamma^2 E \\ &\rightarrow \nabla^2 H = \gamma^2 H \end{aligned}\right] \leftarrow \text{Helmolz equation.}$$

$$\rightarrow \gamma = \sqrt{j\mu\omega(\sigma + j\omega\epsilon)} \leftarrow \text{propagation constant.}$$

$$\rightarrow \alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$$

$$\rightarrow \beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}$$

$$\rightarrow \beta = \frac{2\pi}{\lambda}, \quad \lambda = \frac{c}{f}, \quad \omega t - \beta z = \text{Const.}$$

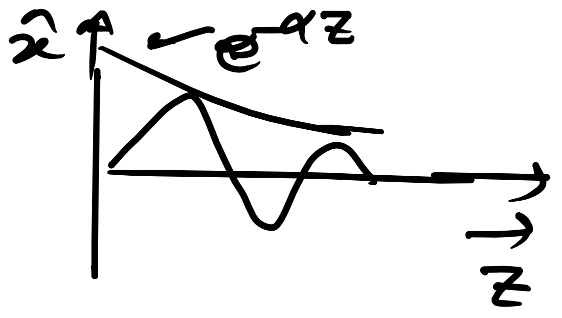
$$\frac{\omega}{\beta} = v_p$$

$$\rightarrow v_p = \frac{c}{\sqrt{\mu_r \epsilon_r}} =$$

e

$$\rightarrow \vec{E} = \mathcal{E}_0 e^{-\alpha x} \cos(\omega t - \beta z)\hat{x}$$

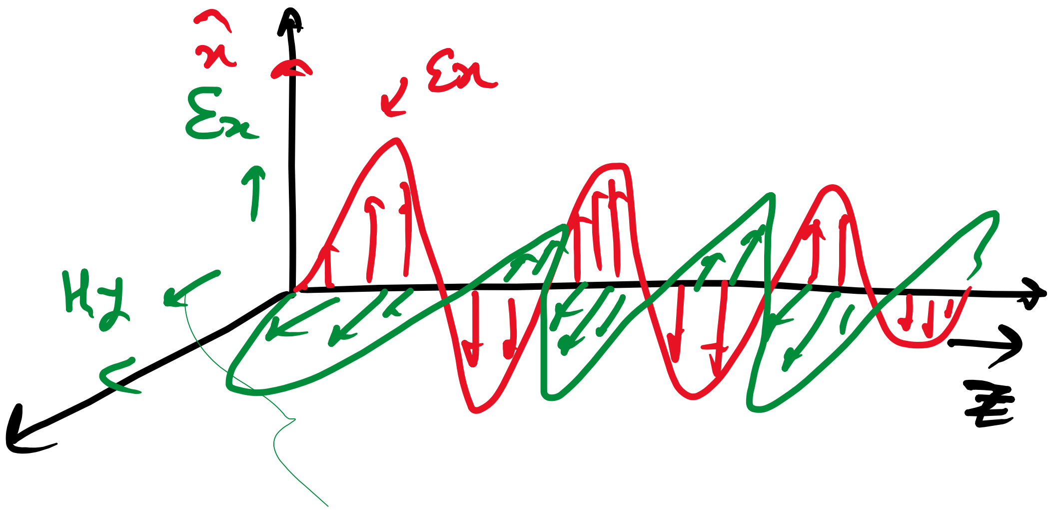
$$\rightarrow \mathcal{E} = \mathcal{E}_0 e^{+-\gamma z} e^{j\omega t} + \mathcal{E}_0 e^{-\gamma z}$$

→ 

$$\rightarrow \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\vec{E}(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\,\hat{x} \quad \text{—— (1)}$$

z — space / distance; t — time



→ Electromagnetic wave is transverse in nature.

→ $\vec{E}$, $\vec{H}$ & direction of propagation will perpendicular to each other

$$H_y = H_0 e^{-\alpha z} \cos(\omega t - \beta z)\,\hat{y} \quad \text{—— (2)}$$

$$\boxed{H_0 = \frac{E_0}{\eta}}$$

η → wave impendance or Intrinsic impedance.

$$\nabla \times E = -j\omega\mu H \quad — ①$$

$$\rightarrow \mathcal{E} = \mathcal{E}_0 e^{-\gamma z}\hat{x} = \mathcal{E}_x$$

$$\mathcal{E} \in f(z)$$

$$\nabla \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \mathcal{E}_x & \mathcal{E}_y & \mathcal{E}_z \end{vmatrix}$$

$$\nabla \times E = \hat{x}\left[\cancel{\frac{\partial \mathcal{E}_z}{\partial y}}^{0} - \frac{\partial \mathcal{E}_y}{\partial z}\right] - \hat{y}\left[\cancel{\frac{\partial \mathcal{E}_x}{\partial z}}^{0} - \frac{\partial \mathcal{E}_x}{\partial z}\right] + \hat{z}\left[\cancel{\frac{\partial \mathcal{E}_y}{\partial x}}^{0} - \cancel{\frac{\partial \mathcal{E}_x}{\partial y}}^{0}\right] = -j\omega\mu(\hat{x}H_x + \hat{y}H_y + \hat{z}H_z)$$

$$-\hat{x}\frac{\partial \mathcal{E}_y}{\partial z} = -j\omega\mu H_x \hat{x} \quad — ②$$

$$+\hat{y}\frac{\partial \mathcal{E}_x}{\partial z} = -j\omega\mu H_y \hat{y} \quad — ③$$

from ③

$$\frac{\partial \mathcal{E}_x}{\partial z} = -j\omega\mu H_y$$

$$\rightarrow -\gamma \mathcal{E}_0 e^{-\gamma z} = -j\omega\mu H_y$$

$$-\gamma E_x = -j\omega\mu H_y$$

$$\rightarrow \frac{\mathcal{E}_x}{H_y} = \frac{j\omega\mu}{\gamma} \quad — ④$$

fun ②

$$\frac{\mathcal{E}_y}{H_y} = -\frac{j\omega\mu}{\gamma} \quad — ⑤$$

$$\frac{E_x}{H_y} = \frac{j\omega\mu}{\gamma} = \eta \qquad \frac{E_y}{H_x} = -\frac{j\omega\mu}{\gamma} = -\eta$$

$$\eta = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}$$

$$\boxed{\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}} \longrightarrow \textcircled{c} \leftarrow \text{complex number.}$$

$$\eta = \eta_r + j\eta_I = |\eta| e^{j\theta_\eta}$$

$$\eta = \frac{E_0}{H_0} = \frac{E_x}{H_y} = |\eta| e^{j\theta_n}$$

$$E_x = |\eta| e^{j\theta_n} H_y.$$

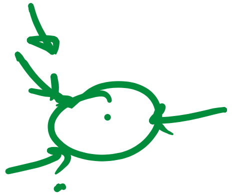
$$\rightarrow \quad E = E_0 e^{-\gamma z} \cos(\omega t - \beta z)\,\hat{x}$$

$$H = E_0 e^{-\gamma z} \cos(\omega t - \beta z - \theta_n)\,\hat{y}$$

magnetic field will always lag by Electric field by θ_n depending ~~upon~~ the medium

→ α ← unit → Neper/m ⇒ Np/m or in dB

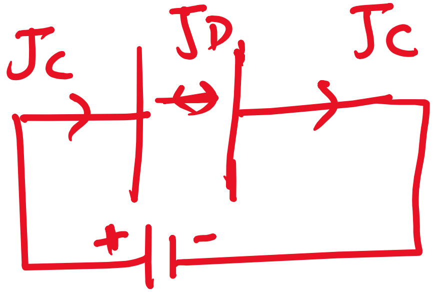
$$1\ \text{Np} = 8.686\ \text{dB}$$



$$J_D = \frac{\partial D}{\partial t} = j\omega\epsilon E$$

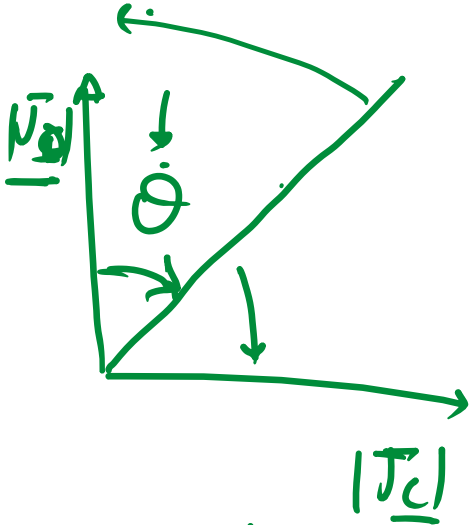
→ β ← phase shift per unit lenght

→ Loss tangent →



$$\tan\theta = \frac{|J_c|}{|J_D|} = \frac{|\sigma E|}{|j\omega\epsilon E|}$$

$$\boxed{\tan\theta = \frac{\sigma}{\omega\epsilon}}$$



$J_D = 0 \Rightarrow \theta = \pi/2$

$J_c = 0 \Rightarrow \theta = 0°$

loss Angle → $\boxed{\theta = 2\theta_n}$

$\tan\theta$ → tells about the losses occured in medium.

$$\nabla \times H = \vec{J}_c + j\omega\epsilon E = \sigma E + j\omega\epsilon E$$

$$\underline{\sigma_c = j\omega\epsilon}\left(1 - j\frac{\sigma}{\omega\epsilon}\right)$$

$$= j\omega\epsilon\left(\frac{\sigma}{j\omega\epsilon} + 1\right)E$$

$$\underline{\nabla \times H} = \boxed{j\omega\epsilon\left[\underline{1 - j}\frac{\sigma}{\epsilon\omega}\right]}E$$

$$\underbrace{j\omega\epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right)}$$

$\sigma_c \leftarrow$ complex conductivity.

Complex permittivity $\rightarrow \underline{\epsilon_c} = \epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right)$

$$\nabla \times H = \sigma_c E \Rightarrow \vec{J}$$

$$\epsilon_c = \epsilon' - j\epsilon''$$

$$\frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon} = \tan\theta \leftarrow \text{loss tangent}$$

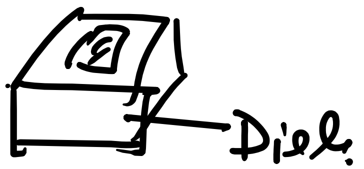
$$\boxed{\tan\theta \propto \epsilon''}$$

$\epsilon' = \epsilon$ —— (A)

$\epsilon'' = \frac{\epsilon\sigma}{\omega\epsilon}$ —— (B)

→ lossy dielectrics ← $\sigma \neq 0$, $\epsilon = \epsilon_r \epsilon_0$, $\mu = \mu_0$, $\mu_r = 1$

$\sigma^2 E$ ← Conductor loss



→ lossless dielectrics → perfect dielectrics → $\sigma = 0$, $\left.\begin{matrix}\mu = \mu_0 \\ \mu_r = 1\end{matrix}\right\}$, $\epsilon = \epsilon_r \epsilon_0$

→ free space → $\sigma = 0$, $\left.\begin{matrix}\mu = \mu_0 \\ \mu_r = 1\end{matrix}\right\}$, $\left.\begin{matrix}\epsilon = \epsilon_0 \\ \epsilon_r = 1\end{matrix}\right\}$

→ perfect conductor → $\sigma \approx \infty$, $\mu = \mu_0 \mu_r$; $\cancel{\mu_r = 1}$ $\epsilon = \epsilon_0$, $\epsilon_r = 1$

Any medium is defined by σ, μ, ϵ

$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

$\eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\epsilon}}$

$\rightarrow \alpha = \omega\sqrt{\dfrac{\mu\epsilon}{2}\left[\sqrt{1+\left(\dfrac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$

$\rightarrow \beta = \omega\sqrt{\dfrac{\mu\epsilon}{2}\left[\sqrt{1+\left(\dfrac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}$

$\beta = \dfrac{2\pi}{\lambda}$

$\rightarrow u_p = \dfrac{\omega}{\beta}$

$E_x = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\hat{x}$

$H_y = H_0 e^{-\alpha z}\cos(\omega t - \beta z - \theta_n)\hat{y}$

$\tan\theta = \dfrac{\sigma}{\omega\epsilon} = \dfrac{\epsilon''}{\epsilon'}$

Wave in lossy dielectrics.

$\sigma \neq 0$, $\mu = \mu_0$, $\epsilon = \epsilon_0\epsilon_r$

$\mu_r = 1$

$\tan\theta \neq 0 \leftarrow$ losses will occur

$\rightarrow \epsilon_0 = 8.85\times10^{-12}$ F/m

$\rightarrow \mu_0 = 4\pi\times10^{-7}$ H/m

wave in lossless dielectrics –

$\sigma = 0$, $\mu = \mu_0$, $\epsilon = \epsilon_0 \epsilon_r$

$\mu_r = 1$

① $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \sqrt{j^2\omega^2\mu\epsilon} = 0 + j\omega\sqrt{\mu\epsilon} \leftarrow \alpha + j\beta$

$\alpha = 0$, $\beta = \omega\sqrt{\mu\epsilon}$

② $\eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\dfrac{\mu}{\epsilon}} = \dfrac{E_x}{H_y} = \sqrt{\dfrac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} = \eta_0\sqrt{\dfrac{\mu_r}{\epsilon_r}}$

$\uparrow$ free space intrinsic impedance

$\eta = \sqrt{\dfrac{\mu_0}{\epsilon_0}} = \sqrt{\dfrac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}} = 120\pi$

$= 376.67\,\Omega$

$\boxed{\eta = 120\pi}$

(3) $$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}} = \frac{1}{\underline{\sqrt{\mu_0\epsilon_0}}\sqrt{\mu_r\epsilon_r}}$$

$$\frac{1}{\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{8.85\times10^{-12}\times4\pi\times10^{-7}}} = 2.9999\times10^{8}\ \underline{m/s} \approx 3\times10^{8}\ \underline{m/s}.$$

$- c$

Speed of light

$$v_p = \frac{c}{\sqrt{\epsilon_r\mu_r}}$$ If $\underline{\mu_r=1}$

non magnetic medium

lossless dielectrics

$$\Rightarrow \boxed{v_p = \frac{c}{\sqrt{\epsilon_r}}}$$

In loss less dielectrics
E & H remain in
Same Phase.
$\underline{\theta_n = 0^\circ}$

$\underline{\theta_n = 0^\circ} \rightarrow$ $$E_x = E_0 e^{-\alpha z}\cos(\omega t-\beta z)\hat{x}$$
$$H_y = H_0 e^{-\alpha z}\cos(\omega t-\beta z)\hat{y}.$$

Free space → $\sigma = 0$, $\mu = \mu_0$, $\epsilon = \epsilon_0$

$\mu_r = 1$ $\epsilon_r = 1$

→ $\gamma = \omega\sqrt{\mu_0 \epsilon_0}$

→ $\eta = \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi = 376.67\,\Omega$

→ $v_p = c$

→ $\alpha = 0$

→ $\beta = \omega\sqrt{\mu_0 \epsilon_0} = \gamma$

→ $\theta_\eta = 0°$

→ $E_x = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\,\hat{x}$

→ $H_y = \frac{E_0}{\eta_0} e^{-\alpha z} \cos(\omega t - \beta z)\,\hat{y}$.

Plane wave in good Conductor - ← maybe magnetic as well.

$\sigma \simeq \infty$, $\mu = \mu_0 \mu_r$, $\epsilon = \epsilon_0$

$\epsilon_r = 1$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\gamma = \sqrt{j\omega\mu\sigma\left(1 + \cancel{\frac{j\omega\epsilon}{\sigma}}\right)} \quad 0$$

$$\gamma = \sqrt{j\omega\mu\sigma(1+0)}$$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\frac{\sigma}{\omega\epsilon} - \cancel{1}\right]}$$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\frac{\sigma}{\omega\epsilon}} = \omega\sqrt{\frac{\sigma\mu}{2\omega}}$$

$$\alpha = \sqrt{\frac{\sigma\mu\omega}{2}}, \quad \beta = \sqrt{\frac{\sigma\mu\omega}{2}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma(1+0)}} = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{0 + \frac{j\omega\mu}{\sigma}}$$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]} = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{\left(\frac{\sigma}{\omega\epsilon}\right)^2\left\{\cancel{\frac{1}{(\sigma/\omega\epsilon)^2}} + 1\right\}} - 1\right]}$$

$\sigma >> \omega\epsilon$

Perfect conductor - $\alpha = \beta = \sqrt{\frac{\sigma \mu \omega}{2}}$

$\tan\theta_n = \left(\frac{1}{1}\right)$

$\theta_n = 45^\circ$

$\gamma = \sqrt{\frac{\sigma \mu \omega}{2}}\,(1+j)$

$\eta = \sqrt{\frac{\omega \mu}{2}}\,(1+j)$

$\theta_n = 45^\circ$

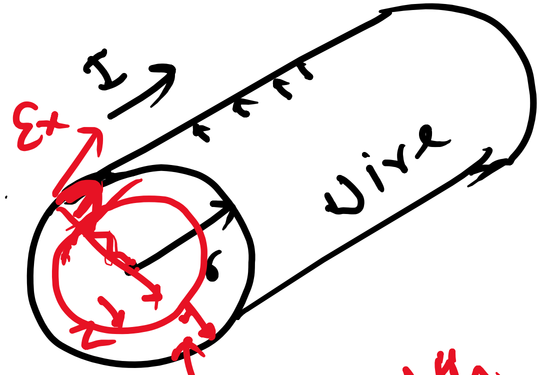
$v_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\sigma \mu \omega}{2}}} = \sqrt{\frac{2\omega}{\sigma \mu}}$

$\lambda = \frac{2\pi}{\beta}$

→ $E_x = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\,\hat{x}$

→ $H_y = H_0 e^{-\alpha z} \cos(\omega t - \beta z - 45^\circ)\,\hat{y}$

→ Conductors → Skin depth.



$\boxed{z = \delta}$

$$E_0 e^{-\gamma z} = E_0 e^{-\gamma \delta}$$

$$E_0 e^{-\alpha z} = E_0 e^{-\alpha \delta} = E_0 e^{-1}$$

$$\alpha \delta = 1$$

$$\boxed{\delta = \frac{1}{\alpha}} \leftarrow \text{Skindepth}$$

$$E = E_0 e^{-1}$$

$$\boxed{E = \frac{E_0}{e^1}} = 36.67\% E_0$$

$$\boxed{E = 0.3677 E_0}$$

$$\alpha = \sqrt{\frac{\sigma \omega \mu}{2}} \qquad \omega = 2\pi f$$

$$\delta = \sqrt{\frac{2}{\sigma \mu \omega}}$$

$$\delta = \sqrt{\frac{2}{2\pi f \sigma \mu}}$$

$$\delta = \frac{1}{\sqrt{\pi f \sigma \mu}}$$

Power & Poynting Vector →

$\rightarrow \nabla \times E = -j\omega\mu H$ —— (1)

$\rightarrow \nabla \times H = \sigma E + j\omega\epsilon E$ —— (2)

from-(2)

$E\cdot(\nabla\times H) = \sigma E^2 + \epsilon \frac{\partial E}{\partial t}\cdot E$

$E\cdot(\nabla\times H) = \sigma E^2 + E\cdot\epsilon\frac{\partial E}{\partial t}$

$E\cdot(\nabla\times H) = \sigma E^2 + \frac{\epsilon}{2}\frac{\partial E^2}{\partial t}$ —— (3)

vector entity.

$\nabla\cdot(E\times H) = H\cdot(\nabla\times E) - E\cdot(\nabla\times H)$

$E\cdot(\nabla\times H) = -\nabla\cdot(E\times H) + H\cdot(\nabla\times E)$ —— (4)

$H\cdot\frac{\partial H}{\partial t} = \frac{1}{2}\frac{\partial H^2}{\partial t}$; $E\cdot\frac{\partial E}{\partial t} = \frac{1}{2}\frac{\partial E^2}{\partial t}$.

$H\cdot(\nabla\times E) - \nabla\cdot(E\times H)$

$= \sigma E^2 + \frac{\epsilon}{2}\frac{\partial E^2}{\partial t}$

$-H\mu\frac{\partial H}{\partial t} - \nabla\cdot(E\times H)$

$= \sigma E^2 + \frac{\epsilon}{2}\frac{\partial E^2}{\partial t}$

$-\frac{\mu}{2}\frac{\partial H^2}{\partial t} - \nabla\cdot(E\times H)$

$= \sigma E^2 + \frac{\epsilon}{2}\frac{\partial E^2}{\partial t}$

$-\nabla\cdot(E\times H) = \sigma E^2 + \frac{\epsilon}{2}\frac{\partial E^2}{\partial t} + \frac{\mu}{2}\frac{\partial H^2}{\partial t}$

$$\nabla\cdot(E\times H) = -\sigma E^2 - \left(\frac{\epsilon}{2}\frac{\partial E^2}{\partial t} - \frac{\mu}{2}\frac{\partial H^2}{\partial t}\right)$$

$$\sigma E^2 = \sigma E\cdot E = \int_v J\cdot E\, dv$$

Gauss - Divergence theorem. -

$$\int_v \nabla\cdot(E\times H)\, dv = -\int_v \sigma E^2\, dv - \frac{\partial}{\partial t}\int_v \left\{\frac{\epsilon}{2}E^2 + \frac{\mu}{2}H^2\right\} dv$$

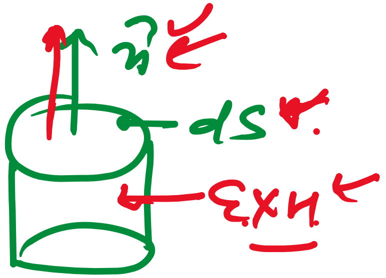
$$\rightarrow \oint_s (E\times H)\cdot \bar{ds} = -\frac{\partial}{\partial t}\int_v \left(\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right) dv - \int_v \sigma E^2\, dv$$

$\oint_s (E\times H)\cdot \bar{ds}$ → Total power → leaving the Volume.

rate of decay of stored energy

Conductor loss

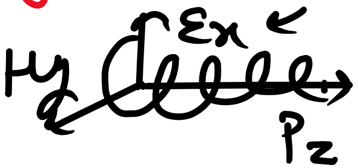
$$\bar{ds} = \hat{n}\, ds$$



$E\times H$ ← Poynting vector

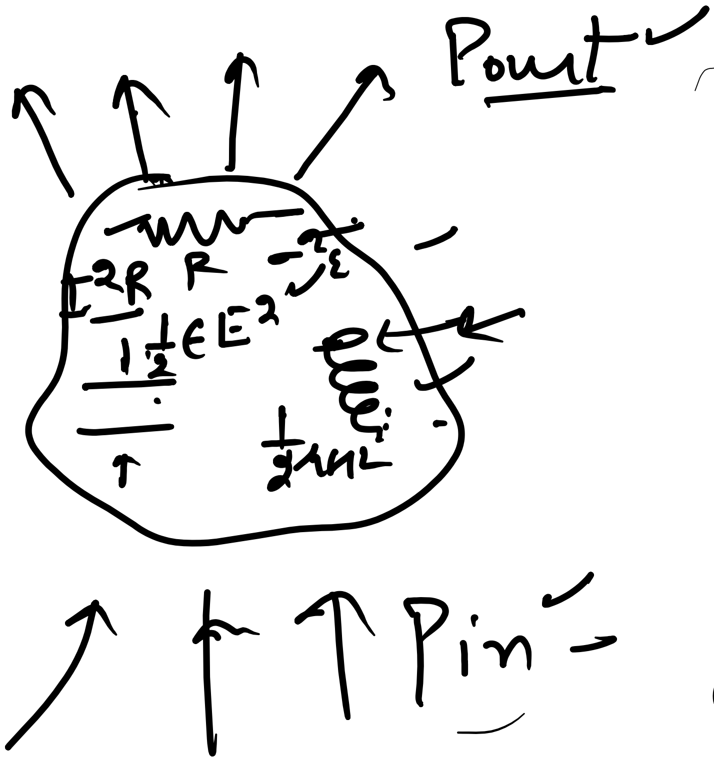
← $\bar{P}$

Poynting theorem states that the total power flowing out of a given volume is equal to the time rate of decay of stored energy within the volume minus Conductor losses.



$$H_0 = \frac{E_0}{\eta}$$

$$P = \underline{E \times H}$$



$$E(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\,\hat{x}$$

$$H(z,t) = H_0 e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\,\hat{y}$$

$$\underline{E \times H} = \frac{E_0^2}{|\eta|} e^{-2\alpha z} \cos(\omega t - \beta z)\cos(\omega t - \beta z - \theta_\eta)\,\hat{z}$$

$$\cos A \cos B = \frac{1}{2}\{\cos(A - B) + \cos(A+B)]$$

$$E \times H = \frac{E_0^2}{2|\eta|}\left[\cos\theta_n + \cos(2\omega t - 2\beta z - \theta_n)\right]\hat{z} = P(z,t)$$

Instantanoas Power.

$T = 2\pi/\omega$

$$\text{Average power} = \frac{1}{T}\int_0^T P(z,t)\cdot dt \Rightarrow P_{av}$$

Powerdensity

$$\boxed{P_{av}(z) = \frac{1}{2}\,\mathrm{Re}\,(E \times H^*)}$$

$$\boxed{P_{av}(z) = \frac{E_0^2}{2|\eta|}\, e^{-2\alpha z}\cos\theta_n\,\hat{z}}$$

Totall time average power :-

$$\boxed{P_{ave.} = \int_S P_{ave}(z) \cdot ds}$$