Chapter-9

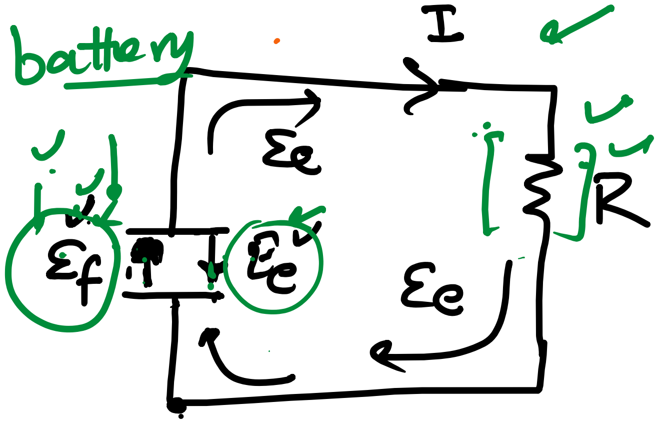
→ Electrostatics → Stationary charge.

→ Magnetostatics → Steady Currents.

→ time varying currents → Electromagnetic waves. or fields.

Faraday's law →

The induced emf, $V_{emf}$ (V) in any closed Path is equal to the time rate of change of flux linkage.




$$V_{emf} = -\frac{d\lambda}{dt} = -\frac{d(N\psi)}{dt}$$

Lenz's law.

The generated emf opposes the Cause of it's generation.. (-ve sign).

$$\boxed{V_{emf} = -\frac{N\,d\psi}{dt}}$$

Total $E = E_f + E_e$ → $E_f \to 0$ → outside the battery.

$\oint E \cdot dl = \oint_l E_f \cdot dl + 0$ → $E_e$ → is opposite inside battery.

(through battery)

$\oint E \cdot dl = \oint_l E_f \cdot dl$.

$\oint E_e \cdot dl = 0 \rightarrow \nabla \times E = 0$

Conservation of field.

$V_{emf}$ $\oint E_f \cdot dl$ $\boxed{= -\oint E\, dl}$ $= IR$

$E_f = -E_e$

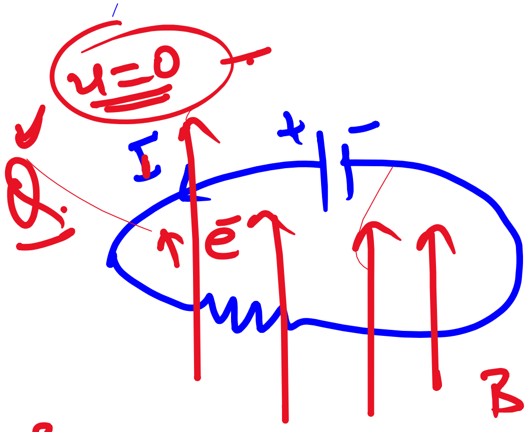
$\psi = \int_s B\cdot ds$

$V_{emf} = -\frac{d\lambda}{dt} = \oint_L E\cdot dl = -N\frac{d\psi}{dt} = -N\frac{d}{dt}\oint B\cdot ds$

$= -N\int \frac{\partial B}{\partial t}\cdot ds$

① Transformer & motional emf →

① Loop is stationary & field is time varying.




$V_{emf} = \oint E\cdot dl = -\int \frac{\partial B}{\partial t}\cdot ds$

$N=1$

$\int_s (\nabla\times E)\cdot ds = -\int \frac{\partial B}{\partial t}\cdot ds$

Time varying field

$B = B_0 e^{j\omega t}$

$\frac{\partial B}{\partial t} \neq 0$

$B \propto f(t)$

Static

$\frac{\partial E}{\partial t} = 0$ & $\frac{\partial B}{\partial t} = 0$

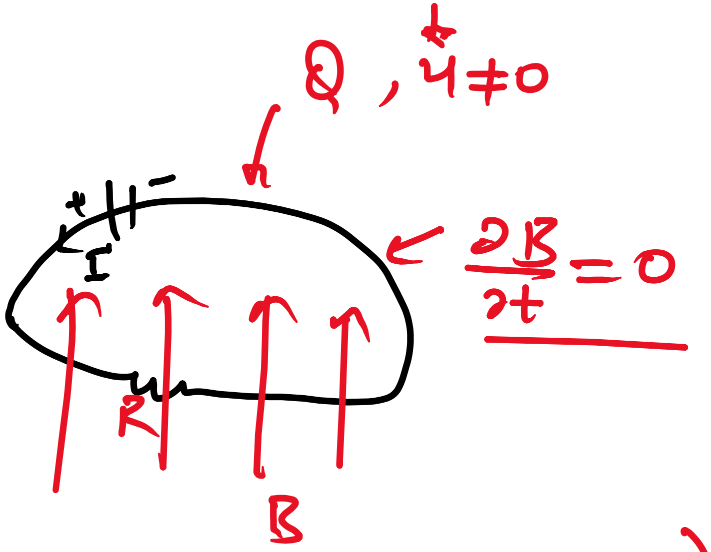
$\frac{\partial B}{\partial t} \neq 0$

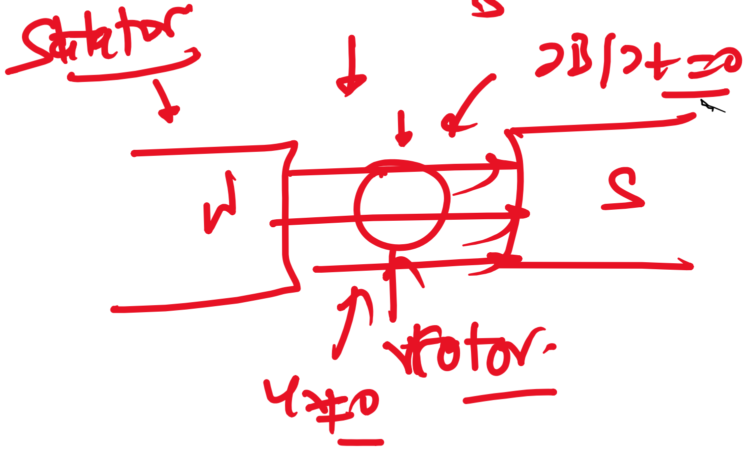
Maxwell's third eqn for time varying field →

$\boxed{\nabla\times E = -\frac{\partial B}{\partial t}}$ ← Stationary charge Faraday's law.

$F = QE + Q\overset{0}{\cancel{u}}\times B$

## ② Moving loop in static field →




$$F = Q\vec{E} + Q\,u \times B$$

$$F_m = Q\,u \times B$$

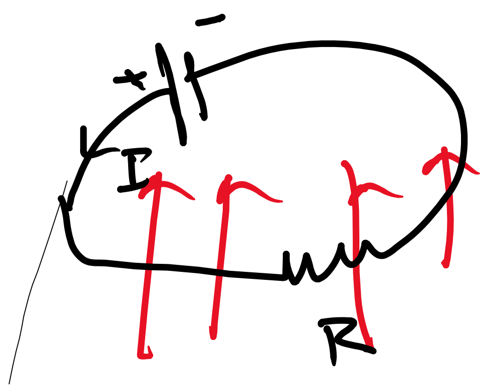
$$\frac{F_m}{Q} = u \times B = E_m$$

$$V_{emf} = \oint_L E_m \cdot dl = \oint_L (u \times B) \cdot dl$$

$$\int_S (\nabla \times E_m)\, ds = \int_S \nabla \times (u \times B)\, ds$$

$$\boxed{\nabla \times E_m = \nabla \times (u \times B)}$$

## (3) Moving loop in time varying field →




$u \neq 0$

$\frac{\partial B}{\partial t} \neq 0$

$$V_{emf} = \oint E \cdot dl = -\int \frac{\partial B}{\partial t} ds + \oint (u \times B) \cdot dl$$

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t} + \nabla \times (u \times B)}$$

## Displament Current →

$$\nabla \times H = J$$

$$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot J$$

→ $\nabla \cdot J \neq 0$

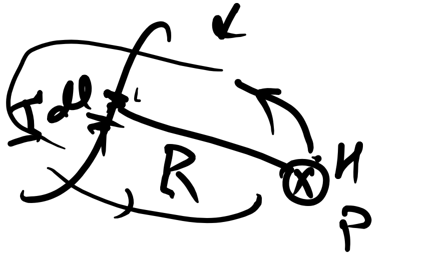
$$\nabla \cdot (\nabla \times \bar{A}) = 0$$

· Displacement current -

$$\nabla \times H = J_c \qquad \nabla \cdot (\nabla \times H) = 0$$

$$\nabla \cdot (\nabla \times H) = \nabla \cdot J_c = 0 \quad \text{— (1)}$$

$$\left\{ \begin{aligned} \nabla \cdot \sigma E &= -\frac{\partial \rho}{\partial t} \\ E &= -\nabla V \\ -\sigma \nabla^2 V &= -\frac{\partial \rho}{\partial t} \\ \nabla^2 V &= -\frac{1}{\sigma}\frac{\partial \rho}{\partial t} \end{aligned} \right.$$




from continuity equation.—

$$I = -\frac{dQ}{dt} = \frac{d\int_v \rho_v \, dv}{dt} = \oint_s J_s \, ds$$

$$\int_v \nabla \cdot J_s \, dv = -\int_v \frac{\partial \rho_v}{\partial t} \, dv$$

$\rightarrow \nabla \cdot J_s = -\frac{\partial \rho_v}{\partial t} \rightarrow$ Continuity equation.

$$\rightarrow \nabla \cdot J_s = -\frac{\partial \rho_v}{\partial t} \neq 0 \quad \text{— (2)}$$

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t} = -\frac{\partial\, \nabla \cdot D}{\partial t} = -\nabla \cdot \left(\frac{\partial D}{\partial t}\right)$$

$$J_c = -\frac{\partial D}{\partial t} \quad \text{--- (3)} \quad \Rightarrow \quad \nabla \cdot J_D = +\nabla \cdot \frac{\partial D}{\partial t}$$

$$\boxed{J_D = \frac{\partial D}{\partial t}} \quad \text{--- (4)}$$

$$\nabla \times H = 0$$

$$\nabla \cdot (\nabla \times H) = 0 \Rightarrow \nabla \cdot J_c + \nabla \cdot J_D = 0$$

$$\nabla \cdot J_c = -\nabla \cdot J_D$$

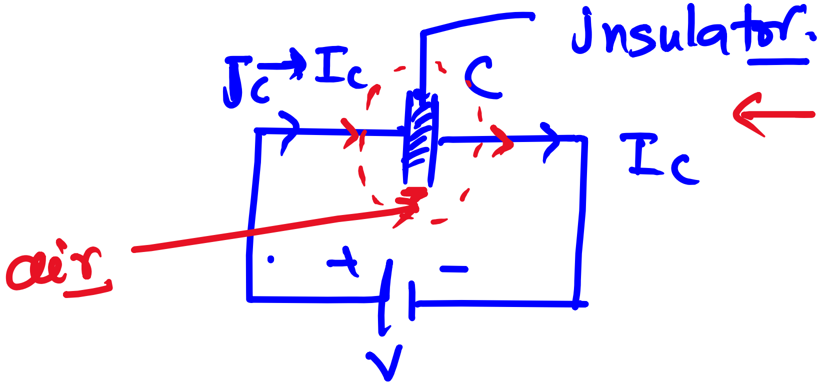
$$\nabla \times H = J + J_D$$

Conduction current density

Displacement current density.

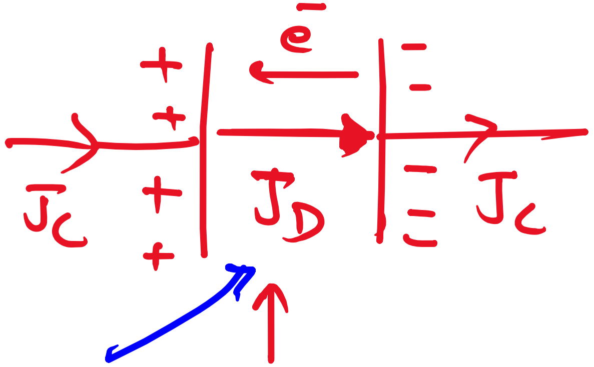
$$\boxed{\nabla \times H = J_c + \frac{\partial D}{\partial t}}$$

← modified Ampere's law.




There is some way in which current flows through the free space or insulator between the parallel plates of the capacitor.

$$\nabla \times H = J_c + \underbrace{\frac{\partial D}{\partial t}}_{J_D}$$




Current can be transmitted in air or free-space

Displacement current.

# Maxwell's equations

→ Time varying field

## Static field

| Point form & differential form | Integral form |
|---|---|
| $\nabla \cdot D = \rho_v$ | $\oint_s D \cdot ds = Q_{en}$ |
| $\nabla \times E = 0$ | $\oint E \cdot dl = 0$ |
| $\nabla \cdot B = 0$ | $\oint B \cdot ds = 0$ |
| $\nabla \times H = J_c$ | $\oint H \cdot dl = I_{en}$ |

Point form
&
differential form.

Integral form

## Point form

$\nabla \cdot D = \rho_v \Rightarrow \oint D \cdot ds = Q_{en}$

$\nabla \cdot B = 0 \Rightarrow \oint B \cdot ds = 0$

$\nabla \times E = -\frac{\partial B}{\partial t}$ ← Faraday's law.

$\nabla \times H = J_c + \frac{\partial D}{\partial t}$ ← modified Ampere's law

$$\oint E \cdot dl = -\int \frac{\partial B}{\partial t} ds$$

$$\oint H \cdot dl = \int_s (J_c + J_D) \cdot ds$$

N, S

$\nabla \cdot B = 0$

$\nabla \times E = 0 \Rightarrow \oint E \cdot dl = 0$ ← Electrostatic field is conservative.

$\nabla \times H = J \neq 0 \Rightarrow$ magnetostatic field is not conservative

→ Time varying E field is not conservation

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \neq 0$$

$\nabla \times \vec{E} \neq 0 \rightarrow$ rotational

$E(t)$ $H(t)$ $E(t)$

$\leftarrow$ Electrodynamo process

$$\nabla \times H = J_c + \frac{\partial D}{\partial t}$$

$\vec{E}$

$D = \epsilon_0 \vec{E}$

$\neq 0$

# Time varying Potentials -

$$B = \nabla \times A$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times E = -\frac{\partial (\nabla \times A)}{\partial t}$$

$$\nabla \times E = -\nabla \times \left(\frac{\partial A}{\partial t}\right)$$

$$\nabla \times \left(E + \frac{\partial A}{\partial t}\right) = 0 \quad \text{— (1)}$$

Scalar

$$\nabla \times (\nabla V) = 0$$

$$\nabla \times \left(E + \frac{\partial A}{\partial t}\right) = -\nabla \times (\nabla V)$$

$$E + \frac{\partial A}{\partial t} = -\nabla V$$

$$\left[ E = -\nabla V - \frac{\partial A}{\partial t} \right] \quad \text{— (2)}$$

$$\nabla \cdot E = -\nabla^2 V - \frac{\partial (\nabla \cdot A)}{\partial t}$$

$$+\frac{\rho_v}{\epsilon} = -\nabla^2 V - \frac{\partial}{\partial t}(\nabla \cdot A)$$

$$\boxed{\nabla^2 V + \frac{\partial}{\partial t}(\nabla \cdot A) = -\frac{\rho_v}{\epsilon}} \quad \text{— (3)}$$

$$B = \mu H \Rightarrow H = B/\mu$$

$$\nabla \times H = J_c + \frac{\partial D}{\partial t} = J_c + \epsilon \frac{\partial E}{\partial t}$$

$$\nabla \times B = \mu J_c + \mu \epsilon \frac{\partial E}{\partial t}$$

$$\nabla \times (\nabla \times A) = \mu J_c + \mu \epsilon \frac{\partial E}{\partial t} \quad \text{— (4)}$$

$$\boxed{\nabla \times (\nabla \times A) = \nabla (\nabla \cdot A) - \nabla^2 A}$$

$$\nabla(\nabla\cdot A) - \nabla^2 A = \mu J_c + \mu\epsilon \frac{\partial E}{\partial t} \quad \text{—} \ (5)$$

from (2) & (5) -

$$\nabla(\nabla\cdot A) - \nabla^2 A = \mu J_c + \mu\epsilon \frac{\partial}{\partial t}\left(-\nabla V - \frac{\partial A}{\partial t}\right)$$

$$\nabla(\nabla\cdot A) - \nabla^2 A = \mu J_c - \mu\epsilon \frac{\partial}{\partial t}(\nabla V) - \mu\epsilon\frac{\partial^2 A}{\partial t^2}.$$

$$\nabla(\nabla\cdot A) = -\mu\epsilon\,\nabla\left(\frac{\partial V}{\partial t}\right)$$

$$\boxed{\nabla\cdot A = -\mu\epsilon\frac{\partial V}{\partial t}} \leftarrow \text{lorentz Condition for potential.}$$

If $\boxed{\nabla\cdot\bar{A} = 0 \leftarrow \text{lorentz Gauge}}$

$$-\nabla^2 A = \mu J_c - \frac{\partial^2 A}{\partial t^2}\mu\epsilon \Rightarrow \boxed{\nabla^2 A = -\mu J_c + \frac{\partial^2 A}{\partial t^2}\mu\epsilon}$$

↑ wave equation.

Time harmonic field →    $\Psi = \underline{E}, \underline{H} \leftarrow f(\underline{t})$

$$\Psi = \Psi_0 e^{j\omega t}$$

$$\mathcal{E} = \underset{\substack{\uparrow \\ \text{magnitude}}}{\mathcal{E}_0}\, e^{j\omega t} \leftarrow \text{Phase}$$

$$\mathcal{E} = \mathcal{E}_0 e^{(j\omega t + j\theta)} = \mathcal{E}_0 \cos(\omega t + \theta) + j\sin(\omega t + \theta)$$

$$\mathcal{E} = \mathcal{E}_0 e^{j\omega t}$$

$$\frac{\partial E}{\partial t} = \mathcal{E}_0 j\omega e^{j\omega t} = j\omega\left(\mathcal{E}_0 e^{j\omega t}\right)$$

$$\boxed{\frac{\partial E}{\partial t} = j\omega \underline{E}}$$

Assignment → Solve all the solved & unsolved numerical problems of chapter-9 of book by Sadiku.