⇒ Poisson's Equation & Laplace Equation -

Bounded media → Capacitor
↓
Boundaries are defined.

in unbounded media →
- Gauss's law → Symmetric
- Coloumb's law → Any
- Scalar Potentials

↑
free space [Dipoles.

→ Poisson's Equation -

$$\nabla \cdot D = \rho_v \Rightarrow \nabla \cdot \epsilon E = \rho_v$$

$$E = -\nabla V \Rightarrow \nabla \cdot \epsilon(-\nabla V) = \rho_v$$

$$\Rightarrow \epsilon \, \nabla \cdot (\nabla V) = -\rho_v$$

$$\boxed{\nabla \cdot (\nabla V) = -\frac{\rho_v}{\epsilon}}$$

For homogenous medium.

$$\nabla\cdot(\nabla A) = \nabla^2 A$$

φ · E
H
F'

$$\Rightarrow \boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}} \leftarrow \text{Poisson's equation.}$$

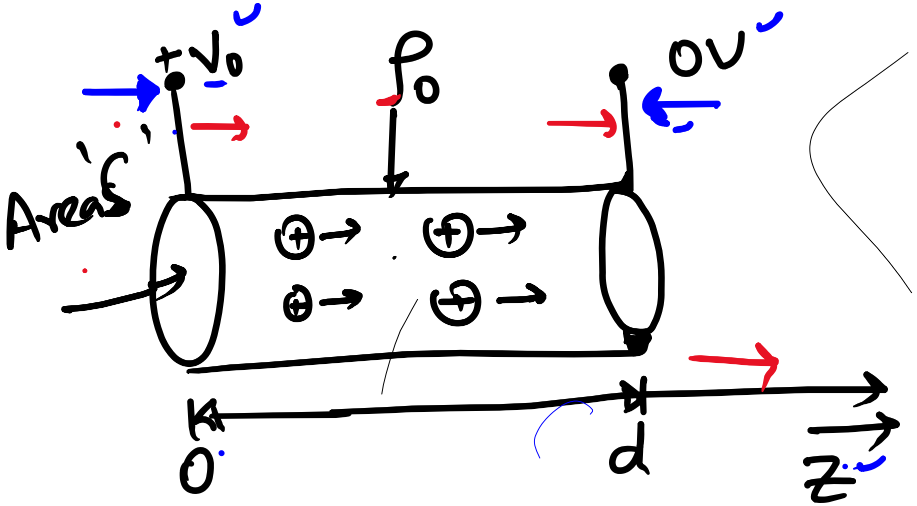
If $\rho_v = 0 \Rightarrow \boxed{\nabla^2 V = 0} \leftarrow$ Laplace's equation.

⇒ <u>Uniqueness Theorem</u> –

To find the solution of any problem we utilize following approaches –
① numerical method (FIT, FEM, FDTD, FDM, BOM, MOM)
② Analytical approach ③ Experimental ④ Graphical.

The solution of Laplace or Poisson equation remains unique under some set of boundary conditions irrespective to the method of finding the solution.

Proof:- Do yourself



Electrohydrodynamic Pumping

Example-61

Calculate the pressure of the pump if $\rho_0 = 25\,mC/m^3$ and $V_0 = 22\,kV$.

$$P = \frac{F}{S} = \frac{Force}{Area} = \frac{QE}{S}$$

$$Q = \int \rho_v \, dv$$

$\rho_v \neq 0$

$$\nabla^2 V = -\frac{\rho_v}{\epsilon}$$

$$\cancel{\frac{\partial^2 V}{\partial x^2}}^{0} + \cancel{\frac{\partial^2 V}{\partial y^2}}^{0} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\epsilon}$$

$$\left[\frac{\partial^2 V}{\partial z^2}\right] = -\frac{\rho_v}{\epsilon} \Rightarrow \boxed{\frac{d^2 V}{dz^2} = -\frac{\rho_v}{\epsilon}} \text{ — (1)}$$

$$E = -\nabla V$$

$$\rightarrow \frac{dV}{dz} = -\frac{\rho_v z}{\epsilon} + A \text{ — (2)}$$

$$V = -\frac{\rho_v z^2}{2\epsilon} + Az + B \text{ — (3)}$$

at $z = 0 \Rightarrow V = V_0$

$V_0 = 0 + 0 + B$

$\boxed{B = V_0}$ — (4)

$z = d \Rightarrow V = 0$

$$0 = \frac{-\rho d^2}{2\epsilon} + Ad + B$$

$$0 = -\frac{\rho d^2}{2\epsilon} + Ad + V_0$$

$$\boxed{A = \frac{\rho d^2}{2\epsilon d} - \frac{V_0}{d} = \frac{\rho d}{2\epsilon} - \frac{V_0}{d}}$$ — (5)

$$Q = \int \rho_v \, dv$$

$$= \int \rho_s \, ds$$

$$\boxed{P = \frac{Q\epsilon}{S} = \frac{\int \epsilon \rho_s \, ds}{S}}$$

⇒ Resistance & capacitance -

$$J = \sigma E$$

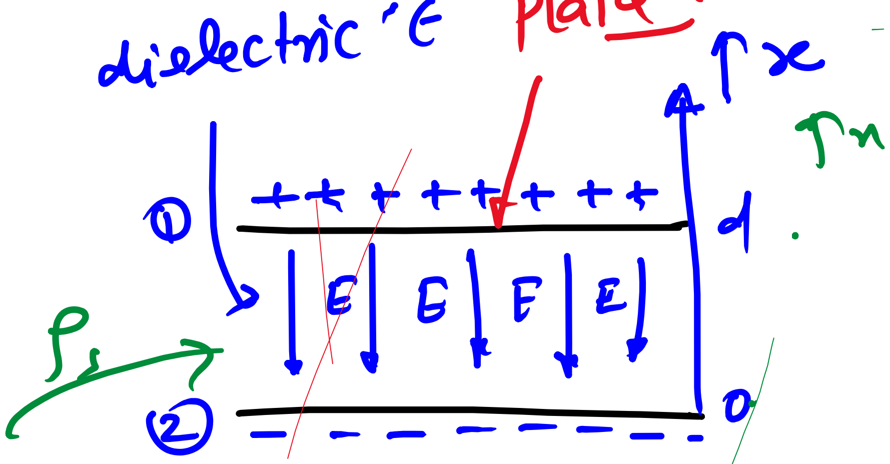
$$R = \frac{V}{I} = \frac{\oint E \cdot dl}{\oint J_s \, ds} = \frac{\oint E \cdot dl}{\oint_s \sigma E \, ds} \quad \text{—— (1)}$$

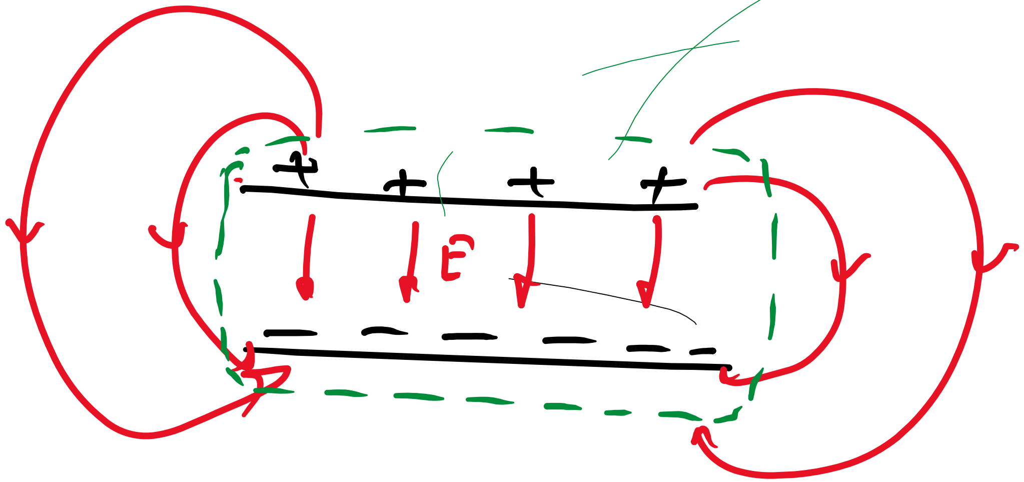
$$C = \frac{Q_{enc}}{V} = \frac{\oint D \cdot ds}{\int E \cdot dl} = \frac{\epsilon \oint E \cdot ds}{\int E \cdot dl} \quad \text{—— (2)}$$

$$RC = \frac{\cancel{\oint E \cdot dl}}{\oint \sigma \cancel{E} \, ds} \cdot \frac{\epsilon \cancel{\oint E} \cdot ds}{\cancel{\int E \cdot dl}} = \frac{\epsilon}{\sigma}$$

$$\boxed{RC = \frac{\epsilon}{\sigma}}$$ ← Time Constants → relaxation time

Parallel Plate capacitor -





$$Q = \oint_s \rho_s \, ds$$

$$C = \frac{Q}{V}$$

$$\rightarrow Q = \rho_s \cdot S$$

$$\Rightarrow \nabla \cdot D = -\rho_s$$

$$\hat{x}\frac{\partial D}{\partial x} + \hat{y}\cancel{\frac{\partial D}{\partial y}}^{0} + \hat{z}\cancel{\frac{\partial D}{\partial z}}^{0} = -\rho_s$$

$$\hat{x}\frac{\partial D}{\partial x} = -\rho_s$$

$$\frac{\partial D}{\partial x} = -\rho_s$$

$$\boxed{D = -\rho_s \hat{x}}$$

$$E = \frac{\rho_s}{\epsilon}(-\hat{x})$$

$$E = -\frac{QS}{\epsilon}(\hat{x})$$

$$V = -\int_L E \cdot dl$$

$$V = -\int_1^2 \left(-\frac{QS}{\epsilon}\right)\hat{x}\, dl$$

$$V = +\int_0^d \frac{QS}{\epsilon}\hat{x}\cdot\hat{x}\,dx$$

$$V = \frac{Qd}{\epsilon S}$$

$$\boxed{\frac{Q}{V} = \frac{\epsilon S}{d} = C}$$

$$CR = \frac{\epsilon}{\sigma}$$

$$R = \frac{\epsilon}{\sigma C} = \frac{\epsilon}{\sigma}\frac{d}{\epsilon S}$$

$$R = \frac{d}{\sigma S} \qquad \sigma = \frac{1}{\rho}$$

$$\boxed{R = \frac{\rho d}{S}}$$

Q. Calculate the capacitor of coaxial cable

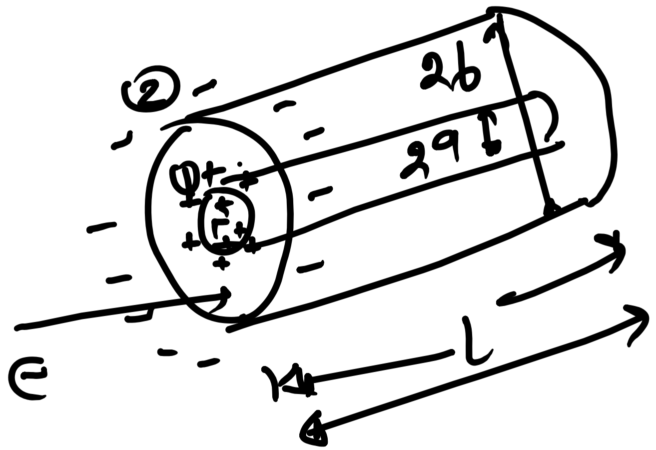
& sphere.

⇒ Coaxial cable -

$C = \frac{Q}{V}$, $Q = \oint D \cdot ds$

$Q = \epsilon \oint E \cdot ds$

$= \epsilon \int_\rho \int_\phi \int_z E \cdot ds$



$ds = d\rho \, \rho d\phi$

$\rightarrow Q = \epsilon \int_\rho \int_\phi \int_z E \cdot \rho d\rho d\phi$

$$\boxed{Q = \epsilon E 2\pi \rho L}$$

$$\Rightarrow \boxed{E = \frac{Q}{2\pi \epsilon \rho L} \hat{\rho}}$$

$V = -\int E \cdot dl$

$= -\int_a^b \frac{Q}{2\pi \epsilon \rho L} \cdot dl$

→ Method of Images →

Q $\quad$ ρ_l $\quad$ ρ_v

Equipotential Surface

$+Q$ $\quad$ ρ_l $\quad$ $+\rho_v$

$-Q$ $\quad$ ρ_l $\quad$ $-\rho_v$

"Perfect electric Conductor" $(\sigma = \infty)$ ← Infinite ground plane.

$+Q$ ← monopole

$V = 0$

$-Q$

$+Q$

Q

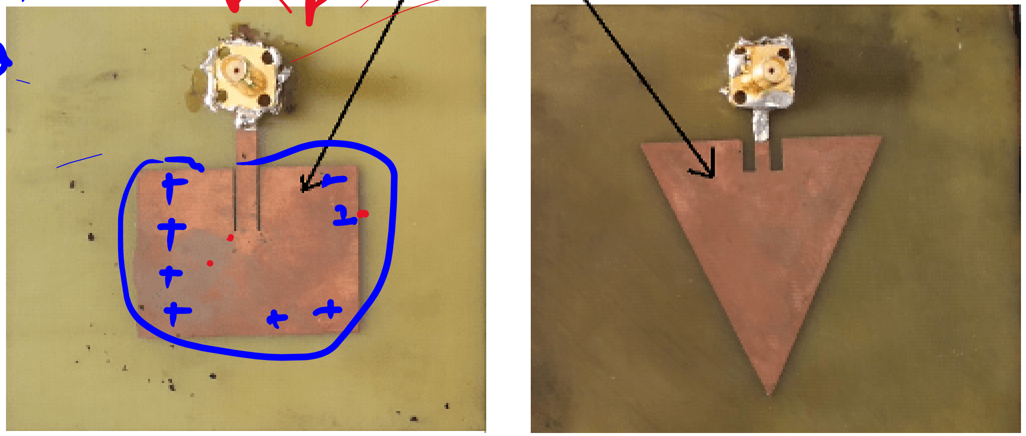
d

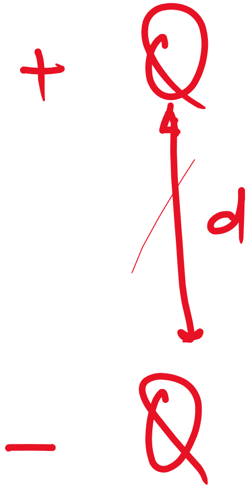
Q

monopole antenna ↳ Can behave as Dipole

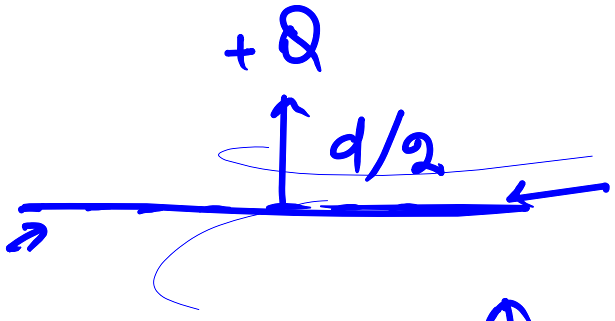
Infinite Ground Plane at back →

Radiating Patch or Shape



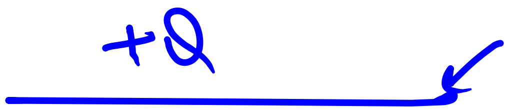


Electric Dipole

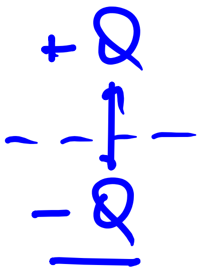


Infinite Ground Plane

Electric monopole



EPS



EPS

Assignment-3 — Solve all the solved & unsolved numerical Problems of Chapter-6 - Sadiku

& solve all the homeworks