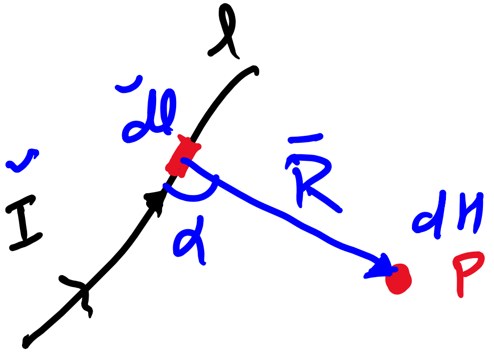
Magnetostatics -

1. Bio savart's law.
2. Ampere's law.
3. vector Poisson Equation
3. magnetic vector Potential

$\bar{A} \times \bar{B} = \bar{C}$

Screwdriver

1. Bio-Savart's law:



$dH \propto I\,dl$

$dH \propto \sin\alpha$

$dH \propto \frac{1}{R^2}$

$dH \propto \frac{I\,dl \sin\alpha}{R^2}$

$K = \frac{1}{4\pi}$

$dH = \frac{I\,dl \sin\alpha}{4\pi R^2}$

$I\,dl \sin\alpha = |I\bar{dl}|\,|\hat{a}_R| \sin\alpha$

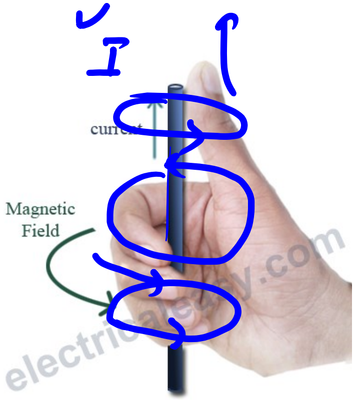
$= I\,\bar{dl} \times \hat{a}_R$

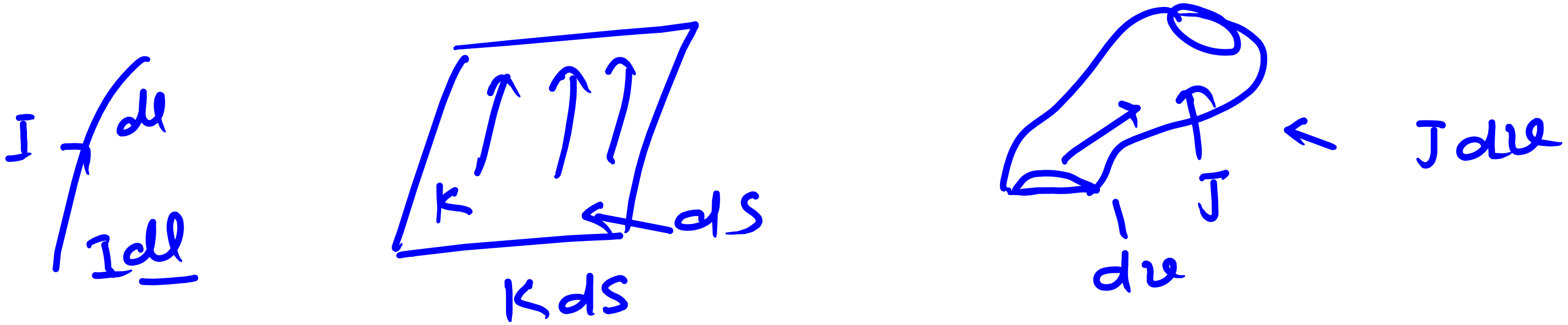
$dH = \frac{I\,dl \times \hat{a}_R}{4\pi R^2}$

$\hat{a}_R = \frac{\bar{R}}{|\bar{R}|}$

$dH = \frac{I\,dl \times \bar{R}}{4\pi R^3}$

$$\boxed{H = \int \frac{I\,dl \times \hat{R}}{4\pi R^3}}$$

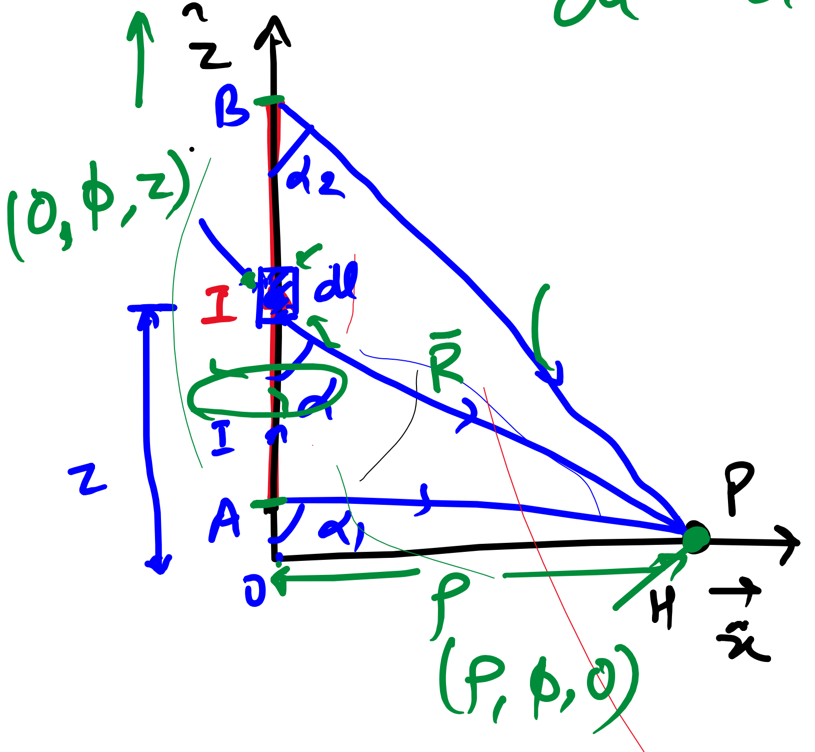




$$H = \int \frac{I\,dl \times \hat{a}_R}{4\pi R^2}$$

$$H = \int \frac{K\,dS \times \hat{a}_R}{4\pi R^2}$$

$$H = \int \frac{J\,d\bar{v} \times \hat{a}_R}{4\pi R^2}$$



$dl = dz\,\hat{z}$ $\quad H = \int \frac{I\,dl \times \hat{a}_R}{4\pi R^2}$ $\quad \hat{a}_R = \frac{\bar{R}}{|\bar{R}|}$

$\bar{R} = (\rho - 0)\hat{\rho} + (\phi - \phi)\hat{\phi} + (0 - z)\hat{z}$

$\bar{R} = \rho\hat{\rho} - z\hat{z}$

$$H = \int \frac{I(\hat{z}\,dz) \times (\rho\hat{\rho} - z\hat{z})}{4\pi(\rho^2 + z^2)^{3/2}}$$

$$dl \times \bar{R} = \begin{vmatrix} \hat{\rho} & \hat{\phi} & \hat{z} \\ 0 & 0 & dz \\ \rho & 0 & -z \end{vmatrix} = \hat{\rho}(0) + \hat{\phi}(\rho\,dz) + \hat{z}(0) = \hat{\phi}\,\rho\,dz$$

$$H = \int \frac{I\,\rho\,dz\,\hat{\phi}}{4\pi(\rho^2 + z^2)^{3/2}}$$

Let $z = \rho \cot\alpha$

$$dz = -\rho \operatorname{cosec}^2\alpha \, d\alpha$$

$$H = \int_{\alpha_1}^{\alpha_2} \frac{-I\rho^2 \operatorname{cosec}^2\alpha \, d\alpha}{4\pi(\rho^2 + \rho^2\cot^2\alpha)^{3/2}} \hat{\phi}$$

$$H = \int -\frac{I \cancel{\rho^2} \cancel{\operatorname{cosec}^2\alpha} \, d\alpha}{4\pi \cancel{\rho^3} \operatorname{cosec}^3\alpha} \hat{\phi}$$

$$H = \int \frac{-I \, d\alpha \, \hat{\phi}}{4\pi\rho \operatorname{cosec}\alpha}$$

$$\boxed{H = -\frac{I}{4\pi\rho} \int_{\alpha_1}^{\alpha_2} \sin\alpha \, d\alpha \, \hat{\phi}}$$

① $l - \to \infty$

$\alpha_1 = 90°, \quad \alpha_2 = 0°$

H_ϕ

$$\boxed{H = \frac{I}{4\pi\rho} \hat{\phi}}$$

② $l \to -\infty$ to ∞

$\alpha_1 = 180°, \quad \alpha_2 = 0°$

$$\boxed{H = \frac{I}{2\pi\rho} \hat{\phi}}$$ ✓

Example → 7.1, 7.2, 7.3, 7.4

Ampere's law - The line integral of tangential Component of H around a closed path is the same as current enclosed by the path.

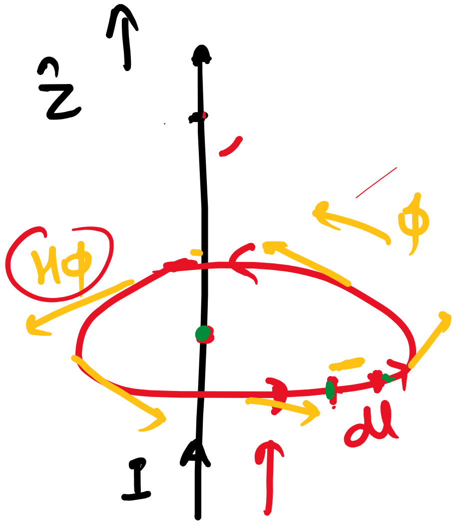
$\int_s D \cdot ds = Q_{enc}$

$$\oint H \, dl = I_{enc} = \int_s J \cdot ds$$

$$\int_s \nabla \times H \, ds = \int_s J \cdot ds$$

$\boxed{\nabla \times H = J}$ ← Ampere's circuital law.

Applications of Ampere's law -



$$I = \oint H \cdot dl = \oint H_\phi \hat{\phi}$$

$$dl = \rho \, d\phi \, \hat{\phi}$$

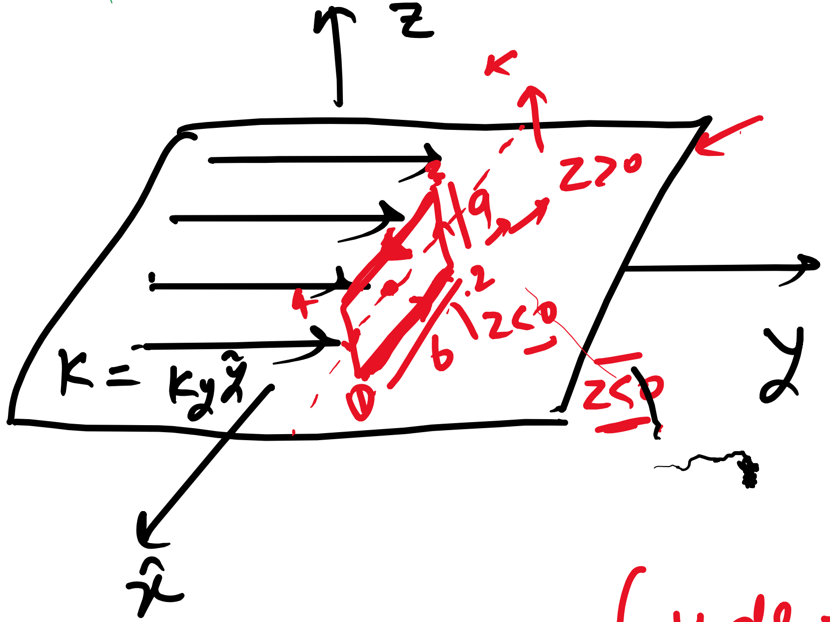
$$I = \oint H_\phi \hat{\phi} \cdot \rho \, d\phi \cdot \hat{\phi}$$

$$I = \int_0^{2\pi} H_\phi \, \rho \, d\phi$$

$$I = H_\phi \cdot 2\pi\rho$$

$$\Rightarrow H_\phi = \frac{I}{2\pi\rho}$$

$$\boxed{H = \frac{I}{2\pi\rho}\hat{\phi}}$$



$$\oint H \cdot dl = I_{enc} = K_y b \quad — ①$$

$$H = \begin{cases} H_0 \hat{x} & z > 0 \\ -H_0 \hat{x} & z < 0 \end{cases} \quad — ②$$

$$\oint H \cdot dl = \left\{ \int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 \right\} H \cdot dl$$

$$= +H_0(b) + 0(a) + H_0(+b) + 0$$

$$= H_0 b + H_0 b = 2H_0 b \quad — ③$$

$$2H_0 b = K_y b$$

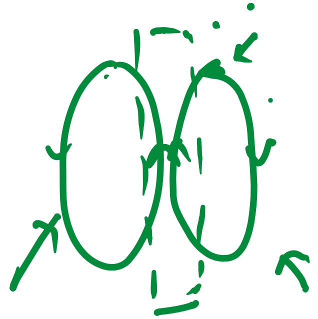
$$\boxed{H_0 = \frac{K_y}{2}}$$

$$\boxed{H = \begin{cases} \frac{K_y}{2} \hat{x} \,; & z > 0 \\ -\frac{K_y}{2} \hat{x} \,; & z < 0 \end{cases}}$$

Calculate the magnetic field in a coaxial wire - H.W.

Magnetic flux density (Maxwell's equation) -

$$\boxed{\nabla\cdot D = \epsilon_0 E}$$



circular

no Source / No Sink

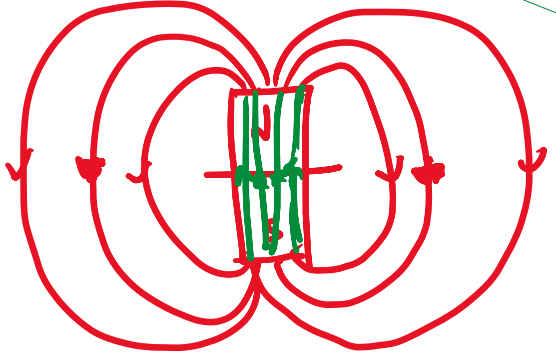
$\mu_0 = 4\pi \times 10^{-7}$ H/m

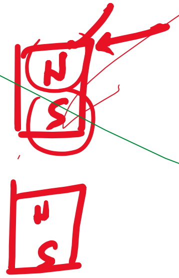
$$\boxed{B = \mu_0 H}$$

$$\boxed{\psi = \oint B\cdot ds}$$

$$\boxed{B = \frac{\psi}{\oint ds}}$$

"Magnetic monopole does not exist"



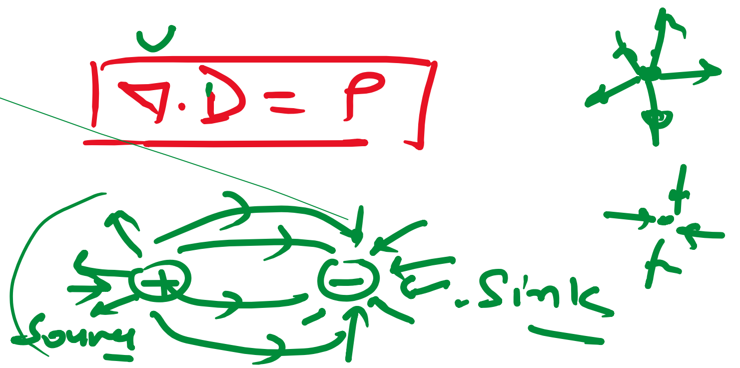


$$\boxed{\nabla\cdot B = 0}$$

↑ maxwell's equation

Divergence -

$$\boxed{\nabla\cdot D = \rho}$$



$\nabla \cdot D = \rho$ Static field

$\nabla \times E = 0$

$\nabla \cdot B = 0$

$\Psi = \oint B \cdot ds$

$\nabla \cdot D = \rho_v$

$\nabla \cdot B = 0$

$\boxed{\nabla \cdot B = 0} \rightarrow$ Gauss's law of magnetostatics.

$\int \nabla \cdot B \, dv = 0$

$= \oint_s \vec{B} \cdot ds = 0$.

Total magnetic flux enclosed by any surface remains zero.

In static fields

1. $\nabla \cdot D = \rho_v$ ← Gauss's law of Electrostatics ← EPS
2. $\nabla \times E = 0$ ← Conservation of electric field. → $E = -\nabla V$ →
3. $\nabla \cdot B = 0$ ← Gauss's law of magnetostatics.
4. $\nabla \times H = J$ ← Ampere's circuital law.

maxwell's equations in static fields. ← Point form or Differential form.

1. $\oint D \cdot dS = Q_{enc}$
2. $\oint E \cdot dl = 0$
3. $\oint B \cdot dS = 0$
4. $\oint H \cdot dl = I_{enc}$

Integral form of maxwell's Equation.

Magnetic Scalar & Vector Potentials-

$\vec{B} = \mu_0 H \leftarrow A/m$

magnetic field intensity

$\nabla \times (\nabla V) = 0$

$\to V \to$ scalar Potential

$\to \nabla \cdot (\nabla \times \bar{A}) = 0$ —(1)

$\bar{A} \to$ Vector Potential

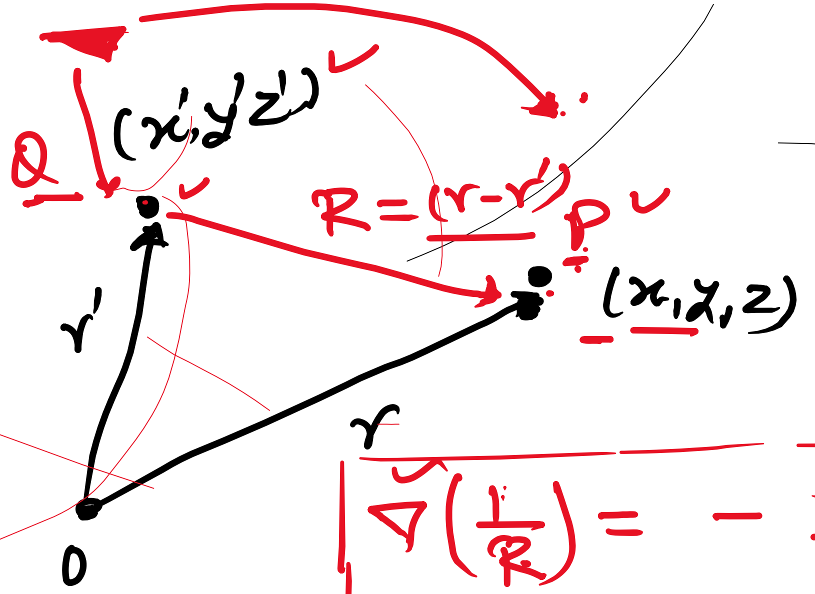
$\nabla \cdot B = 0$ —(2)

$B \to$ magnetic fluxdensity

$B = \mu_0 H$

$$\boxed{\bar{B} = \nabla \times \bar{A}}$$

$\hat{a}_R = \frac{\bar{R}}{|R|}$



$$H = \int_L \frac{I dl \times \bar{R}}{4\pi R^3}$$

$$\boxed{\nabla\left(\frac{1}{R}\right) = -\frac{\hat{a}_R}{R^2}}$$

$$\nabla \times \bar{A} = \bar{B} = \int \frac{\mu_0 I d\bar{l} \times \hat{a}_R}{4\pi \; R^2}$$

$$B = \int \frac{\mu_0 I}{4\pi}\left(\frac{dl' \times \bar{a}_R}{R^2}\right)$$

$$R = (x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}$$

$$\bar{B} = \frac{\mu_0}{4\pi}\int_L \left\{ I\,\bar{dl}' \times \nabla\left(\frac{1}{R}\right)\right\} \quad \text{--- } (1)$$

$f \rightarrow$ Scalar

$\bar{F} \rightarrow$ Vector

$$\nabla \times (f\bar{F}) = f\,\nabla \times F + (\nabla f) \times \bar{F}$$

$$\left[\frac{\bar{Y}.\bar{Z}}{Z} + \frac{\bar{X} \times \bar{F}}{\bar{T}}\right]$$

$$f = \frac{1}{R}$$

$$\bar{F} = dl'$$

$$\nabla \times dl' = 0$$

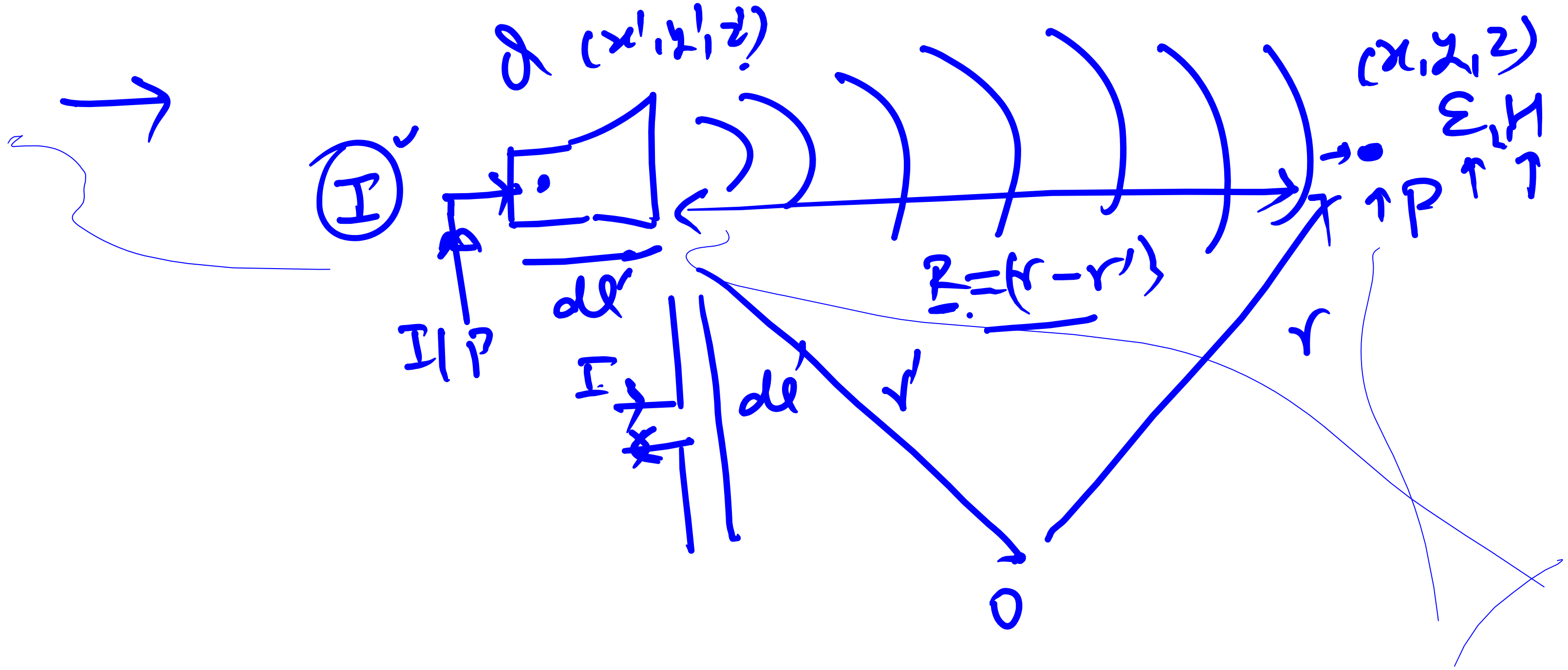
(x, y, z) (x', y', z')

$$\nabla \times (f\bar{F}) = f\,\nabla \times \bar{F} + (\bar{\nabla} f) \times \bar{F}$$

$$\nabla \times \left(\frac{1}{R} dl'\right) = \frac{1}{R}\cancel{\nabla \times dl'} + \left(\nabla \frac{1}{R}\right) \times dl'$$

$$dl' \times \nabla\left(\frac{1}{R}\right) = -\nabla \times \frac{dl'}{R}$$

$$B = \nabla \times \int_L \frac{\mu_0 I\,dl'}{4\pi R} = \nabla \times \bar{A} \Rightarrow \boxed{\bar{A} = \int_L \frac{\mu_0 I\,\bar{dl}'}{4\pi R}}$$



$$\bar{R} = r - r'$$

$$\bar{A} = \int_L \frac{\mu_0 I \, dl'}{4\pi \bar{R}}$$

$$\bar{B} = \nabla \times \bar{A}$$

$$\vec{B} = \mu_0 H$$

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$$

$$\nabla \times \bar{H} = J = \sigma \bar{E}$$

Antenna Engineering.

Assignment → Solve all the solved & unsolved numerical problems of chapter 7

by EMT Sadiku