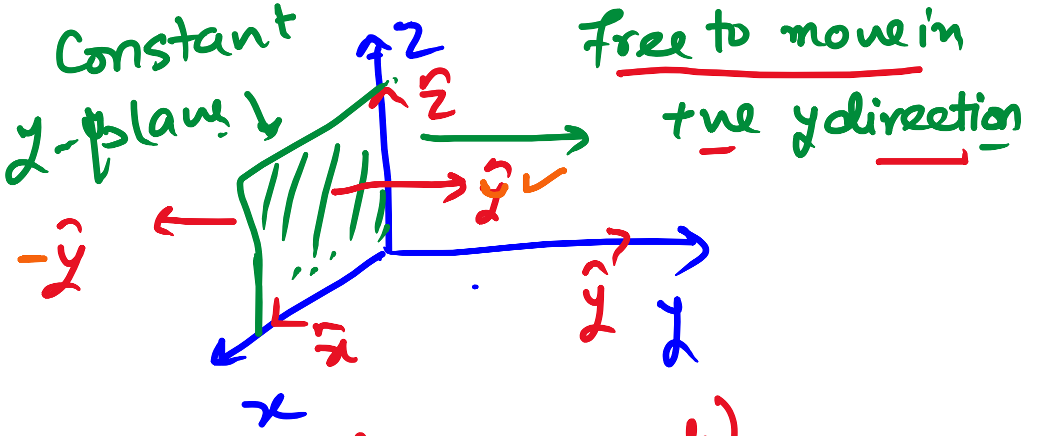
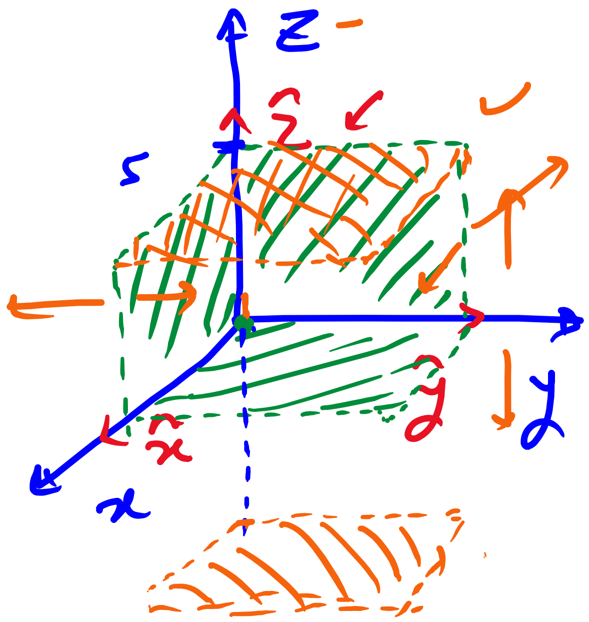
Chapter-2. Coordinate systems.

1. Cartision co-ordinate system:
2. Cylindrical Co-ordinate system
3. Spherical " "



(A unit vector represents the direction of movement)

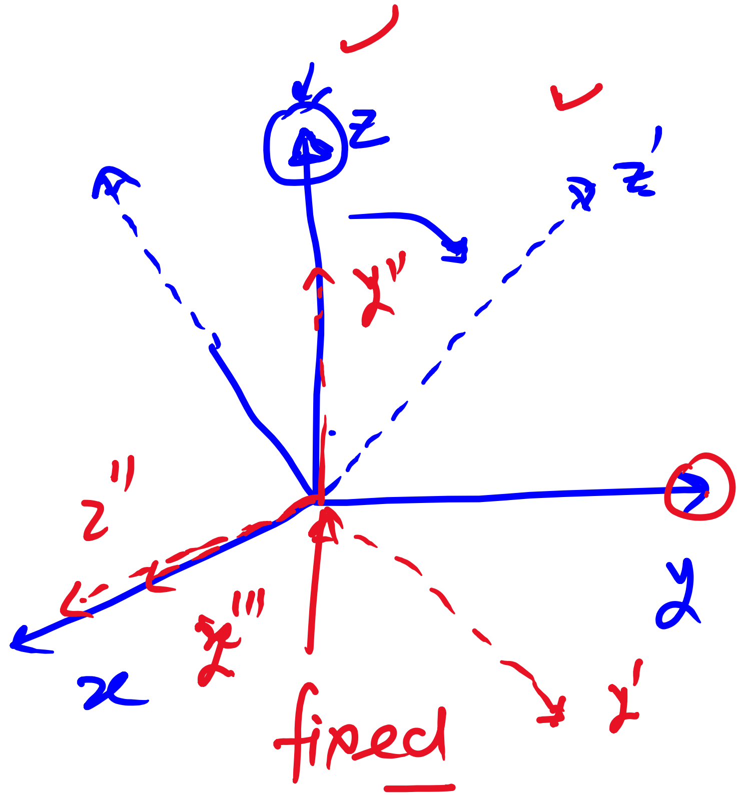
1. Cartision Co-ordinate systems -(rectangular co-ordinates)



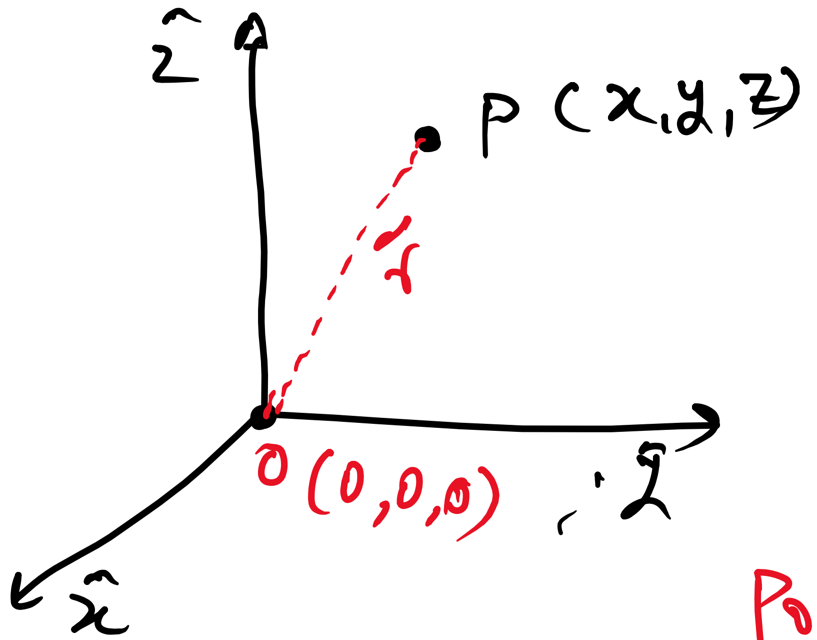
- axis - x, y, z
- unit vectors - $\hat{x}, \hat{y}, \hat{z}$

$$\left\{\begin{array}{ccc} -\infty \leq & x & \leq \infty \\ " & y & " \\ " & z & " \end{array}\right\}$$

- Planes -
  1. $xy \rightarrow z=0 \rightarrow$ Constant-z-Plane.
  2. $yz \rightarrow x=0 \rightarrow$ Constant-x-Plane
  3. $zx \rightarrow y=0 \rightarrow$ Constant-y-Plane.



→ Everything is movable.



Position vector

distance

$$\bar{r} = \bar{P} - \bar{O}$$

$$|\bar{r}| = \sqrt{x^2 + y^2 + z^2}$$

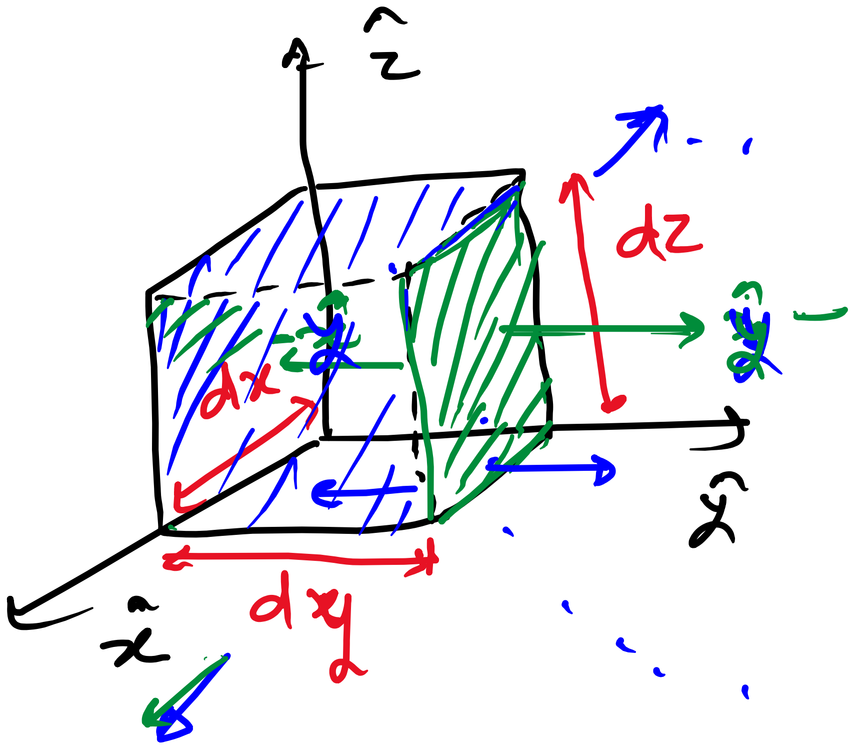
$$\bar{r} = \hat{x}(x-0) + \hat{y}(y-0) + \hat{z}(z-0)$$

$$\boxed{\bar{r} = \hat{x}x + \hat{y}y + \hat{z}z}$$

Distance along axis -

$$d\vec{l} = \hat{x}\,dx$$
$$d\vec{l} = \hat{y}\,dy$$
$$d\vec{l} = \hat{z}\,dz$$

→ length



Area -

$$\hat{y}\,d\vec{A} = \hat{y}\,dx\,dz$$
$$\hat{x}\,d\vec{A} = \hat{x}\,dy\,dz$$
$$\hat{z}\,dA = \hat{z}\,dx\,dy$$

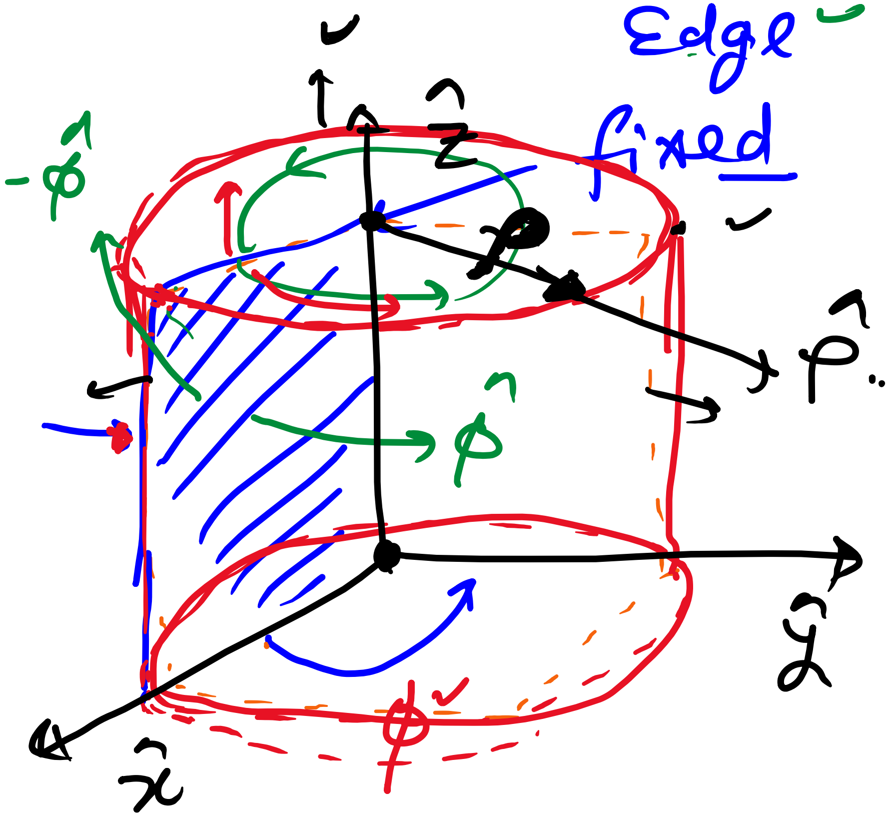
Surface.

Volume -

$$dV = dx\,dy\,dz$$

unit vector is in the direction normal to the surface.

Cylindrical co-ordinate system - (Circular co-ordinates) -



$\phi = 0^\circ \Rightarrow$ xz-Plane

$\phi = 90^\circ \Rightarrow$ yz-Plane

$0 \le \phi \le 2\pi \rightarrow$ draws a cylinder.

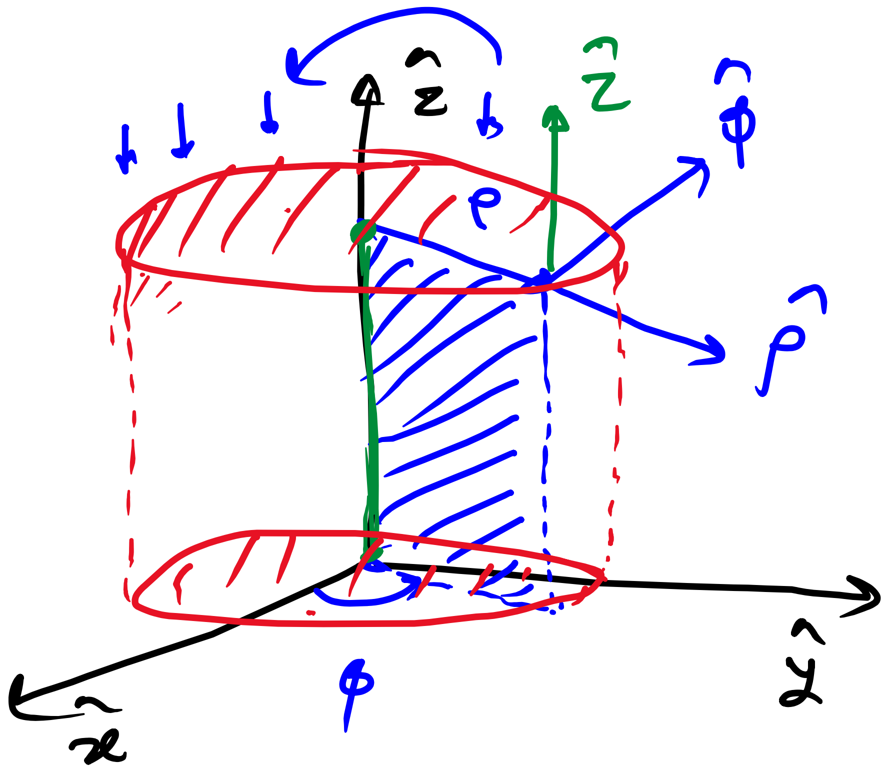
Can be stabilized at perticular value of 'ϕ'.

1. So - we can call it [constant ϕ-Plane.]

unit vector - $\hat{\phi} \rightarrow$

2. Radius of cylinder, $\hat{\rho}$ -

$0 \le \rho \le \infty$



③ height → ($\hat{z}$)

$-\infty \leq z \leq \infty$

unit vectors → $\hat{\rho}$, $\hat{\phi}$, $\hat{z}$

angular

⇒ length - $\hat{\rho}\, d\rho$

$\hat{z}\, dz$

Surface Area → ① Constant ϕ Plane → $\hat{\phi}\, dA$ ⇒ $\hat{\phi}\, ds$

② Top & bottom → $\pm\hat{z}$ $2\pi\rho$..

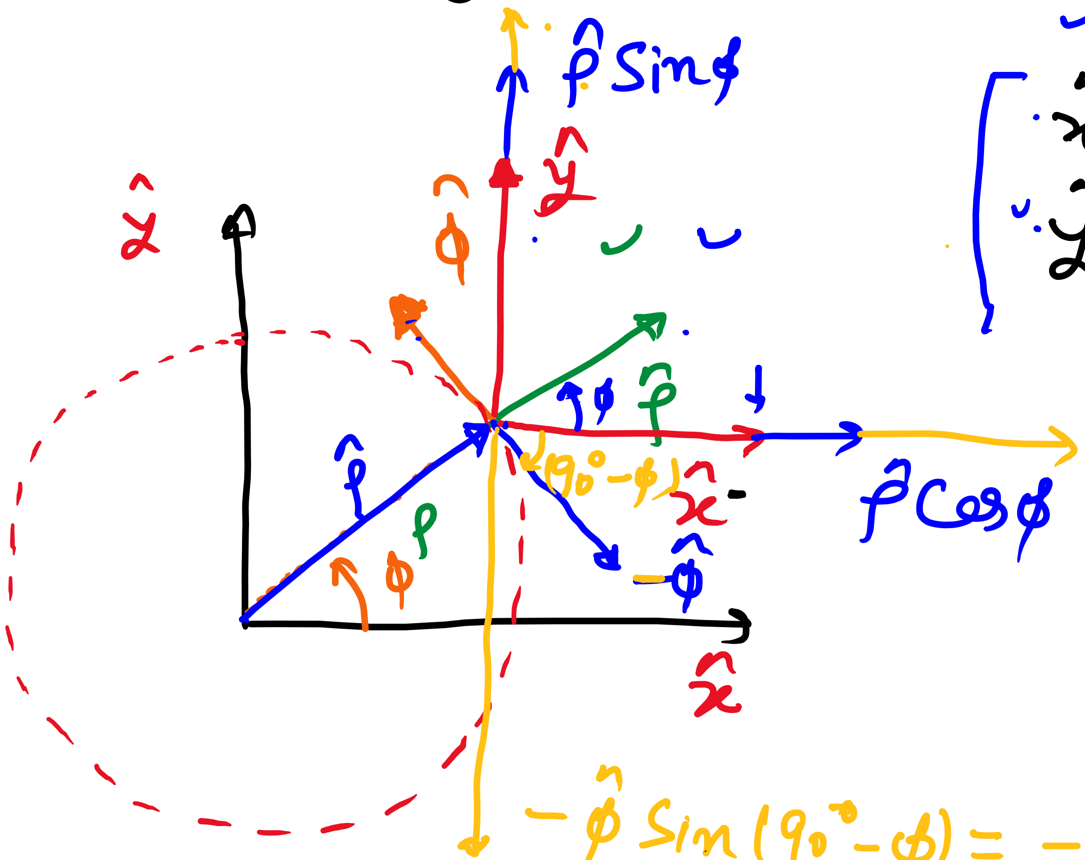
volume - $dv = d\rho\, d\phi\, dz$

Co-ordinate transformations -

$$\text{Cartisian} \underset{②}{\overset{①}{\rightleftarrows}} \text{Cylindrical}$$

$$\bar{A}_{cr} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

$$\bar{A}_{cy} = A_\rho\hat{\rho} + A_\phi\hat{\phi} + A_z\hat{z}$$



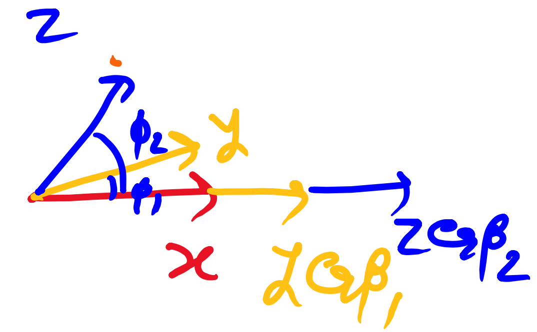
$$\hat{x} = \hat{\rho}\cos\phi + (-\hat{\phi}\sin\phi)$$

$$\hat{y} = \hat{\rho}\sin\phi + \hat{\phi}\cos\phi$$

$$\hat{z} = \hat{z}$$

$$-\hat{\phi}\cos(90^\circ-\phi) = -\hat{\phi}\sin\phi.$$

$$-\hat{\phi}\sin(90^\circ-\phi) = -\hat{\phi}\cos\phi$$



$$x = y\cos\phi_1 + z\cos\phi_2$$

$\Rightarrow \bar{A}_{cy} = \hat{\rho} A_\rho + \hat{\phi} A_\phi + \hat{z} A_z \rightarrow$ Convert in cartisian Co-ordinates.

$\bar{A}_{cr} \rightarrow$ in cartisian coordinates. $\rightarrow$ Convert in cylindrical.

$$\hat{\rho}\cdot\bar{A}_{cy} = \hat{\rho}\cdot\hat{\rho} A_\rho + \hat{\rho}\cdot\hat{\phi} A_\phi + \hat{\rho}\cdot\hat{z} A_z = A_\rho$$

$$\hat{\phi}\cdot\bar{A}_{cy} = \hat{\phi}\cdot\hat{\rho} A_\rho + \hat{\phi}\cdot\hat{\phi} A_\phi + \hat{\phi}\cdot\hat{z}\cdot A_z = A_\phi$$

$$\hat{z}\cdot\bar{A}_{cy} = \hat{z}\cdot\hat{\rho} A_\rho + \hat{z}\cdot\hat{\phi} A_\phi + \hat{z}\cdot\hat{z} A_z = A_z$$

$\rightarrow$ Components of vector along $\hat{\rho}$, $\hat{\phi}$ & $\hat{z}$ axis.

$$A_\rho = \hat{\rho}\cdot\bar{A}_{cr} = \hat{\rho}\cdot A_{cy} = \hat{\rho}\cdot(A_x\hat{x} + A_y\hat{y} + A_z\hat{z})$$

$$A_\rho = \hat{\rho}\,(A_x(\hat{\rho}\cos\phi - \hat{\phi}\sin\phi) + A_y(\hat{\rho}\sin\phi + \hat{\phi}\cos\phi) + \hat{z}\cdot A_z)$$

$$A_\rho = (A_x\cos\phi + A_y\sin\phi + 0) = A_x\cos\phi + A_y\sin\phi \quad \text{--- (1)}$$

$$A_\phi = \hat{\phi} A_{cy} = \hat{\phi} A_{cr} = \hat{\phi} \cdot (A_x \hat{x} + A_y \hat{y} + A_z \hat{z})$$

$$= \hat{\phi} \cdot (A_x (\hat{\rho} \cos\phi - \hat{\phi} \sin\phi) + A_y (\hat{\rho} \sin\phi + \hat{\phi} \cos\phi) + A_z \hat{z})$$

$$A_\phi = -A_x \sin\phi + A_y \cos\phi \quad \text{---} \quad (2)$$

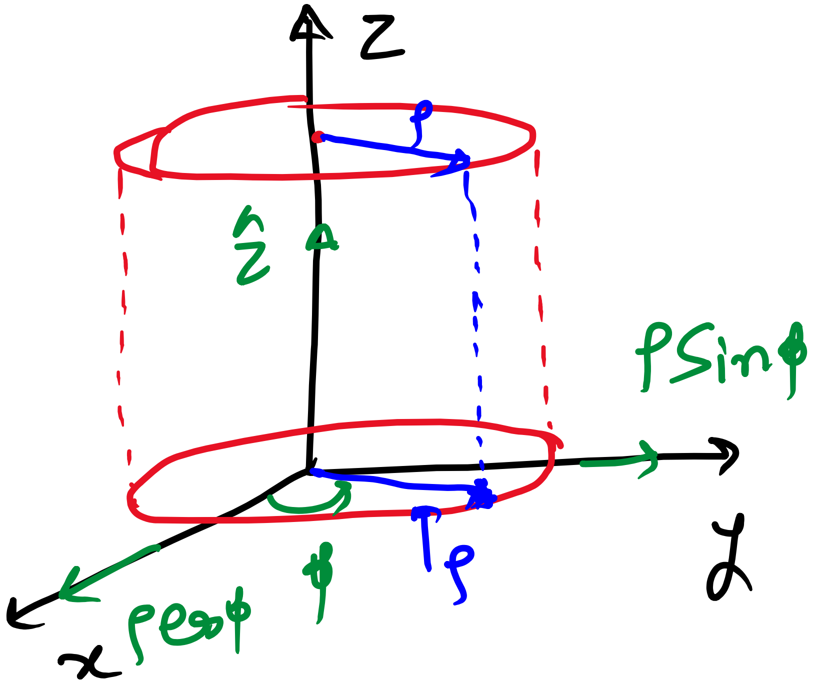
$$A_z = \hat{z} \cdot A_{cy} = \hat{z} A_{cr} = \hat{z} \cdot (\bar{A}_x \hat{x} + A_y \hat{y} + A_z \hat{z}) = A_z \quad \text{---} \quad (3)$$

$$A_\rho = A_x \cos\phi + A_y \sin\phi$$

$$A_\phi = -A_x \sin\phi + A_y \cos\phi$$

$$A_z = A_z$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$



$$\begin{cases} x = \rho\cos\phi \\ y = \rho\sin\phi \\ z = z \end{cases}$$ — Coordinates (Points)

$$\frac{\rho\sin\phi}{\rho\cos\phi} = \frac{y}{x} \Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{y}{x}\right)}$$

Numerical Problem -

Q $P(-2, 6, 3)$ & vector $\bar{A} = \underbrace{y}_{A_x}\hat{x} + \overbrace{(x+z)}^{A_y}\hat{y}$

Express the point & vector in cylindrical coordinates.

Sol →

$P(-2, 6, 3) \Rightarrow x = -2,\ y = 6,\ z = 3.$

$x = \rho\cos\phi$.

$y = \rho\sin\phi$.

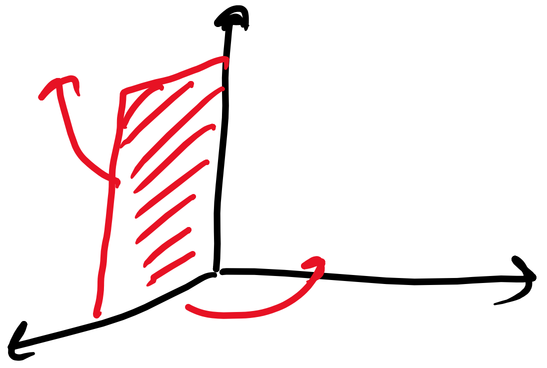
$z = z$

$\Rightarrow \phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{6}{-2}\right) = \tan^{-1}(-3)$

(ρ, ϕ, z)

$\phi = -71.5^\circ$

108.5°



$$\bar{A} = y\hat{x} + (x+2)\hat{y}$$

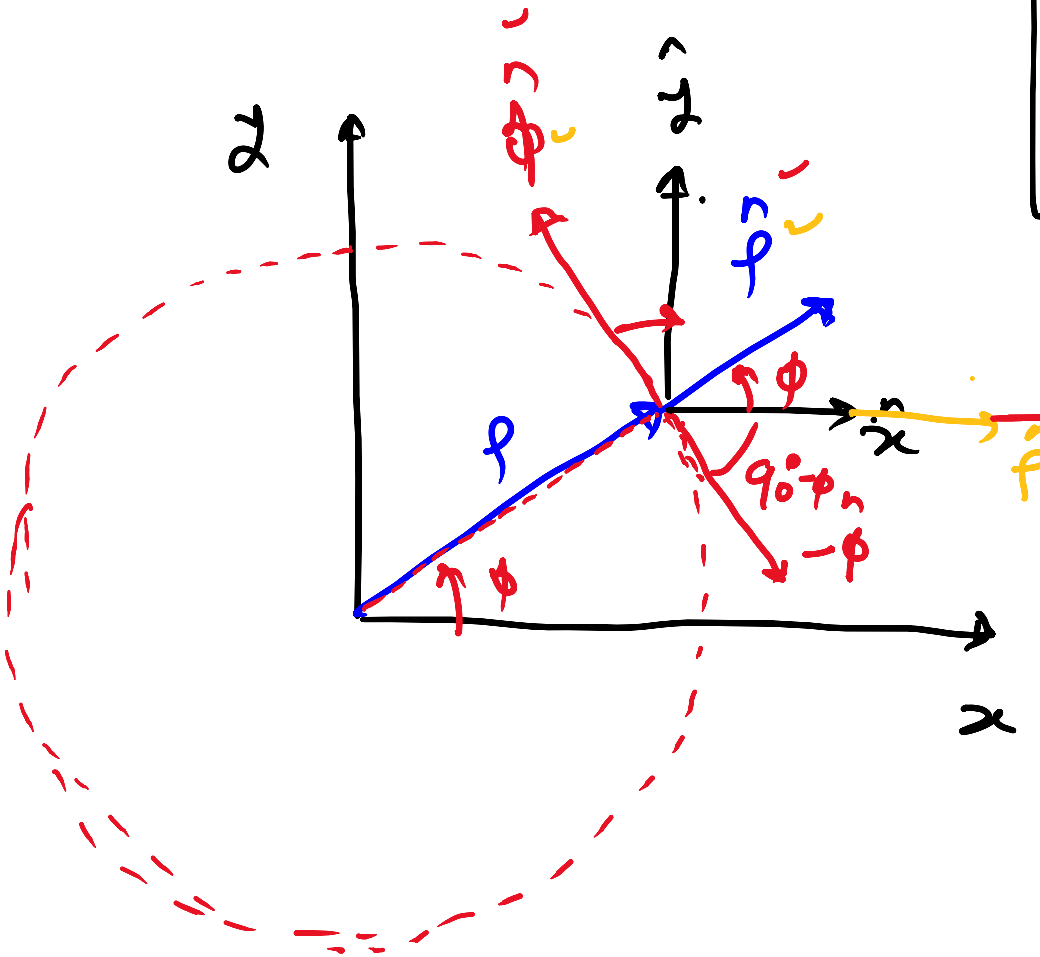
$$A_x = y \ , \ A_y = x+2$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ x+2 \\ 0 \end{bmatrix}$$

[Implement the mathematics of conversion of a vector from cylindrical to Cartisian Co-ordinates.]

Cylindrical to Cartisian →

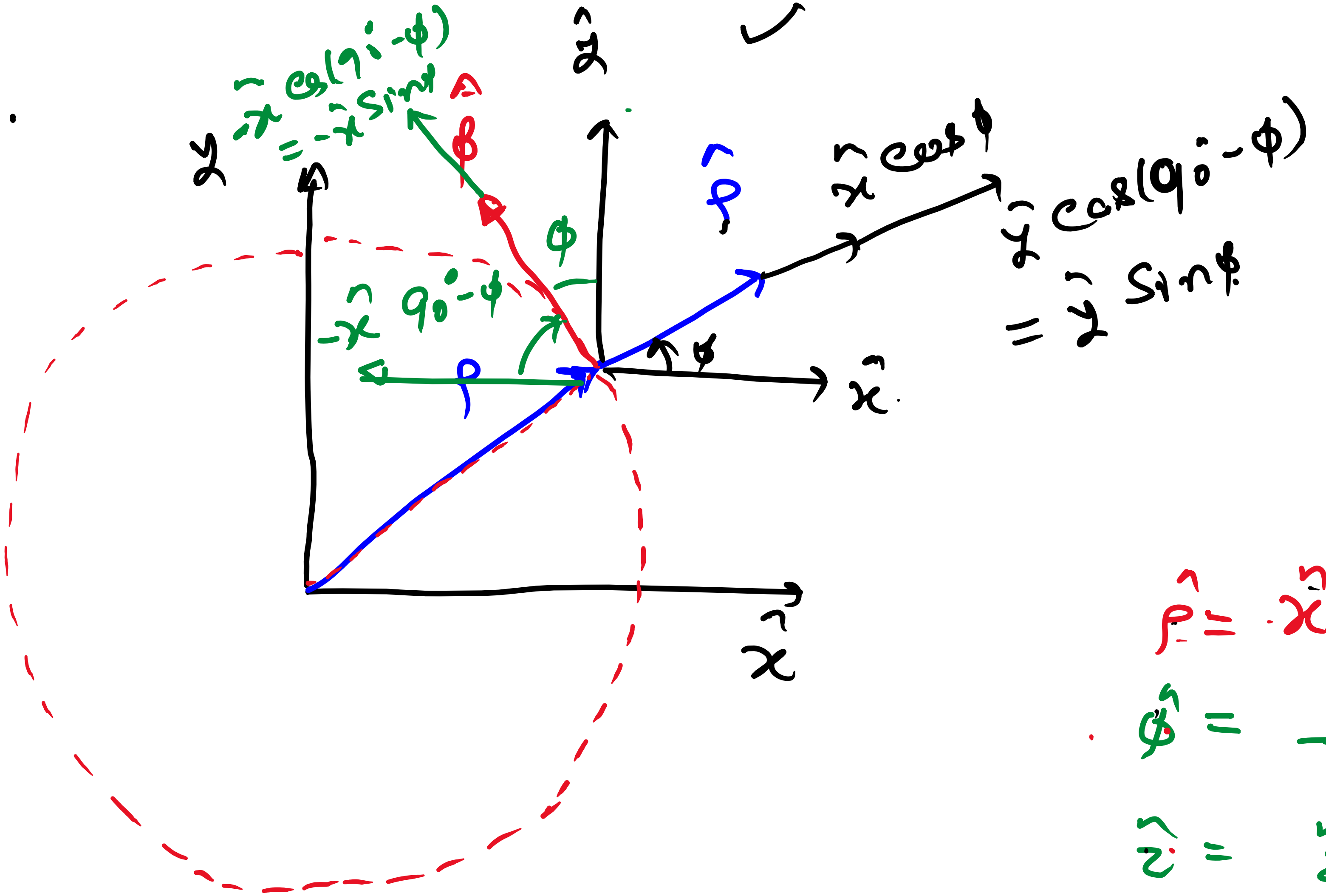
$$\begin{cases} \hat{x} = \hat{\rho}\cos\phi - \hat{\phi}\sin\phi \\ \hat{y} = \hat{\rho}\sin\phi + \hat{\phi}\cos\phi \\ \hat{z} = \hat{z} \end{cases}$$



$\hat{\rho}\cos\phi \quad -\hat{\phi}\cos(90° - \phi)$

$= -\hat{\phi}\sin\phi$

Cartisian to cylindrical →



$$\hat \rho = \hat x \cos\phi + \hat y \sin\phi.$$

$$\hat \phi = -\hat x \sin\phi + \hat y \cos\phi$$

$$\hat z = \hat z$$

Cartisian to cylindrical →

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \hat{\rho}\cdot\hat{x} & \hat{\rho}\cdot\hat{y} & \hat{\rho}\cdot\hat{z} \\ \hat{\phi}\cdot\hat{x} & \hat{\phi}\cdot\hat{y} & \hat{\phi}\cdot\hat{z} \\ \hat{z}\cdot\hat{x} & \hat{z}\cdot\hat{y} & \hat{z}\cdot\hat{z} \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

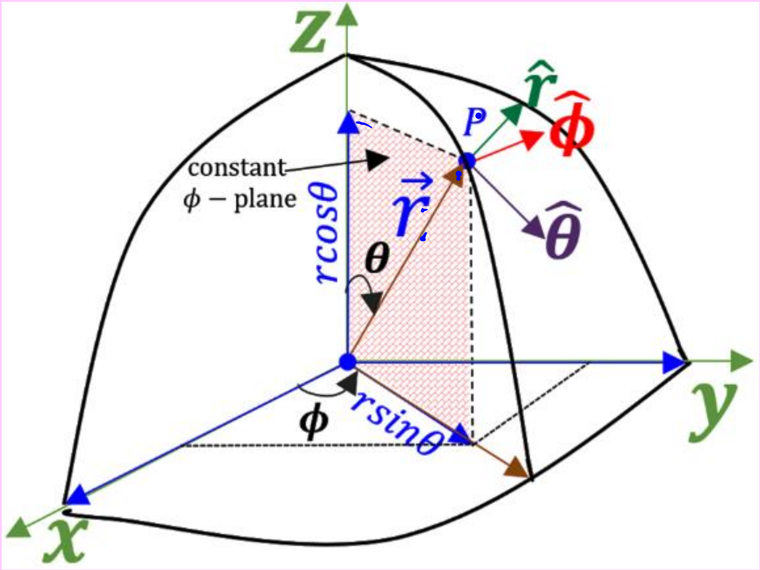
$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

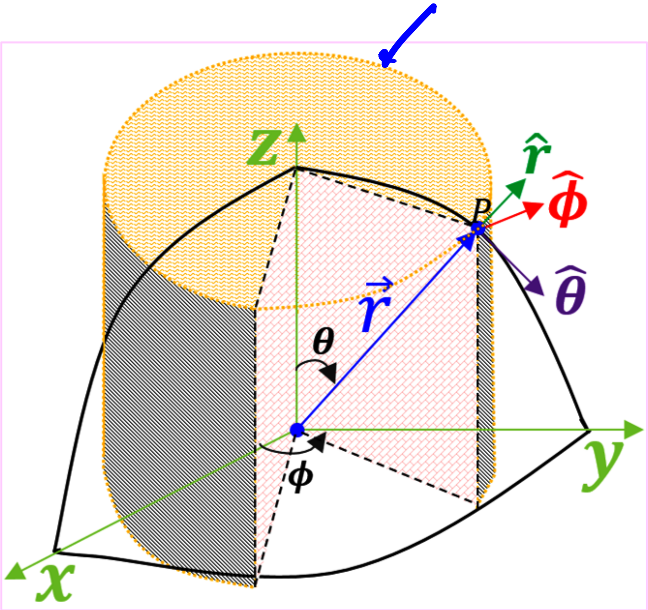
From cylindrical to cartisian $\rightarrow$

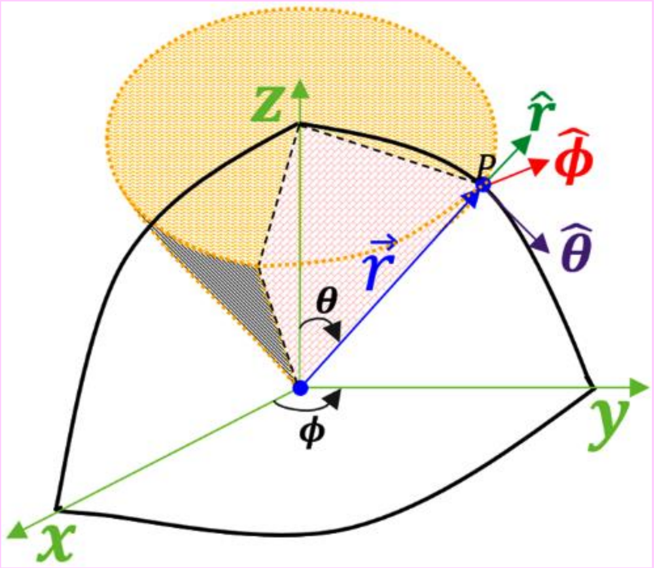
$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \hat{x}\cdot\hat{\rho} & \hat{x}\cdot\hat{\phi} & \hat{x}\cdot\hat{z} \\ \hat{y}\cdot\hat{\rho} & \hat{y}\cdot\hat{\phi} & \hat{y}\cdot\hat{z} \\ \hat{z}\cdot\hat{\rho} & \hat{z}\cdot\hat{\phi} & \hat{z}\cdot\hat{z} \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

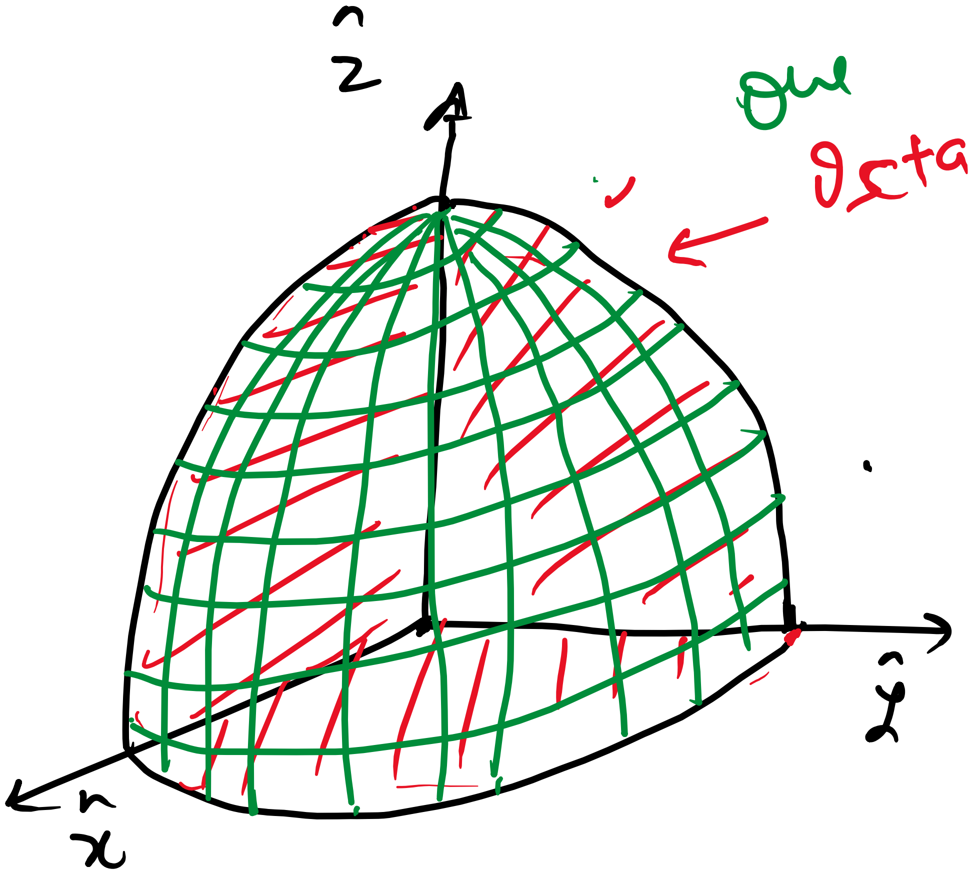
Spherical Co-ordinates -

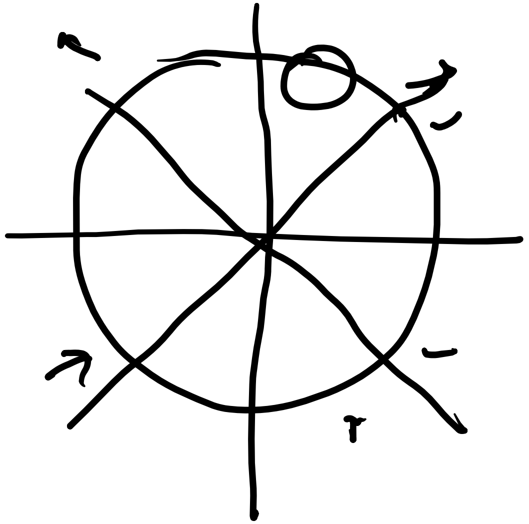


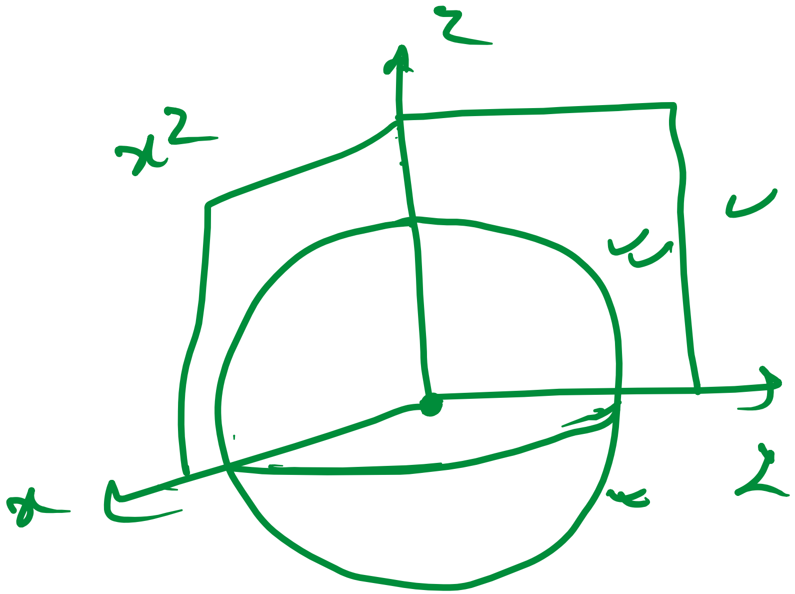




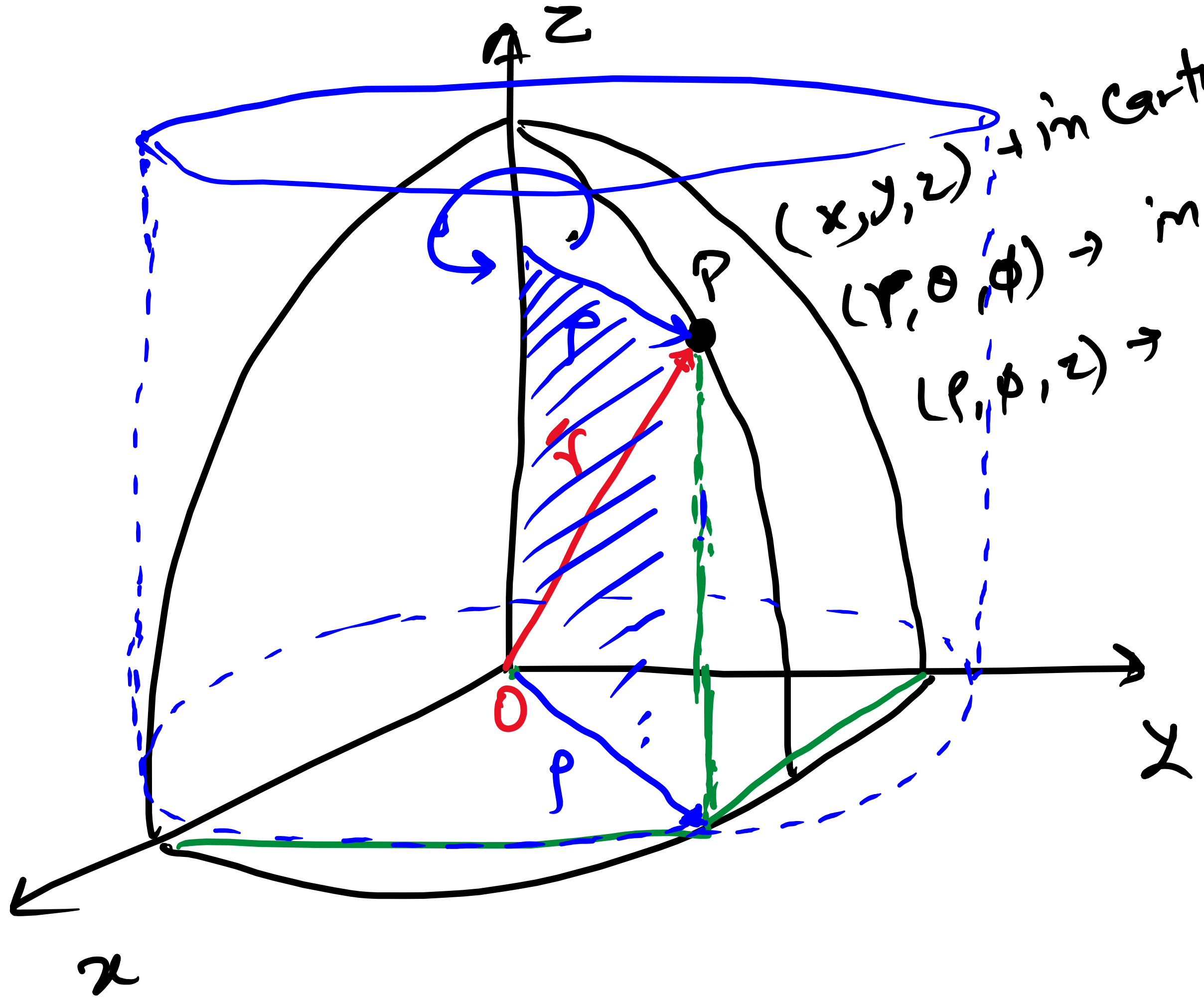
Our octa sphere



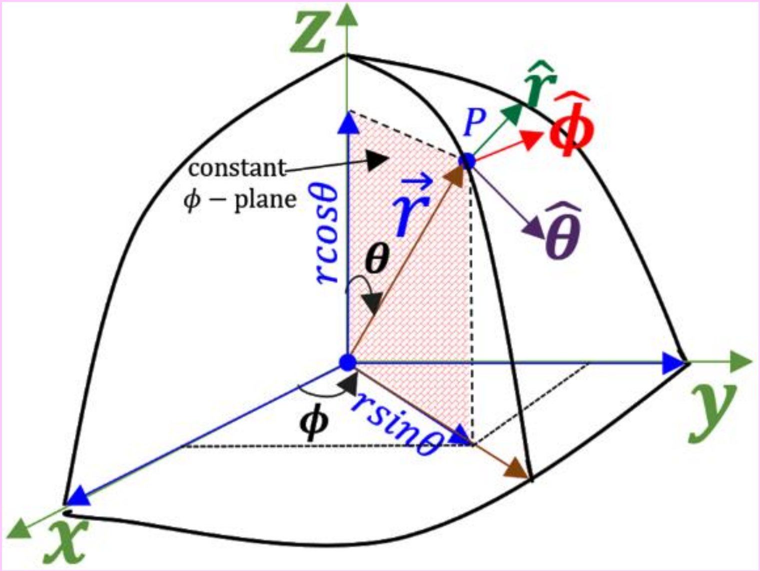


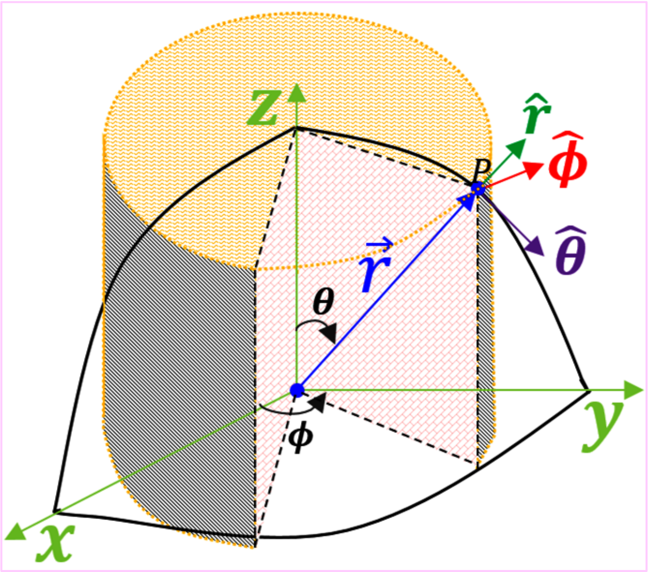


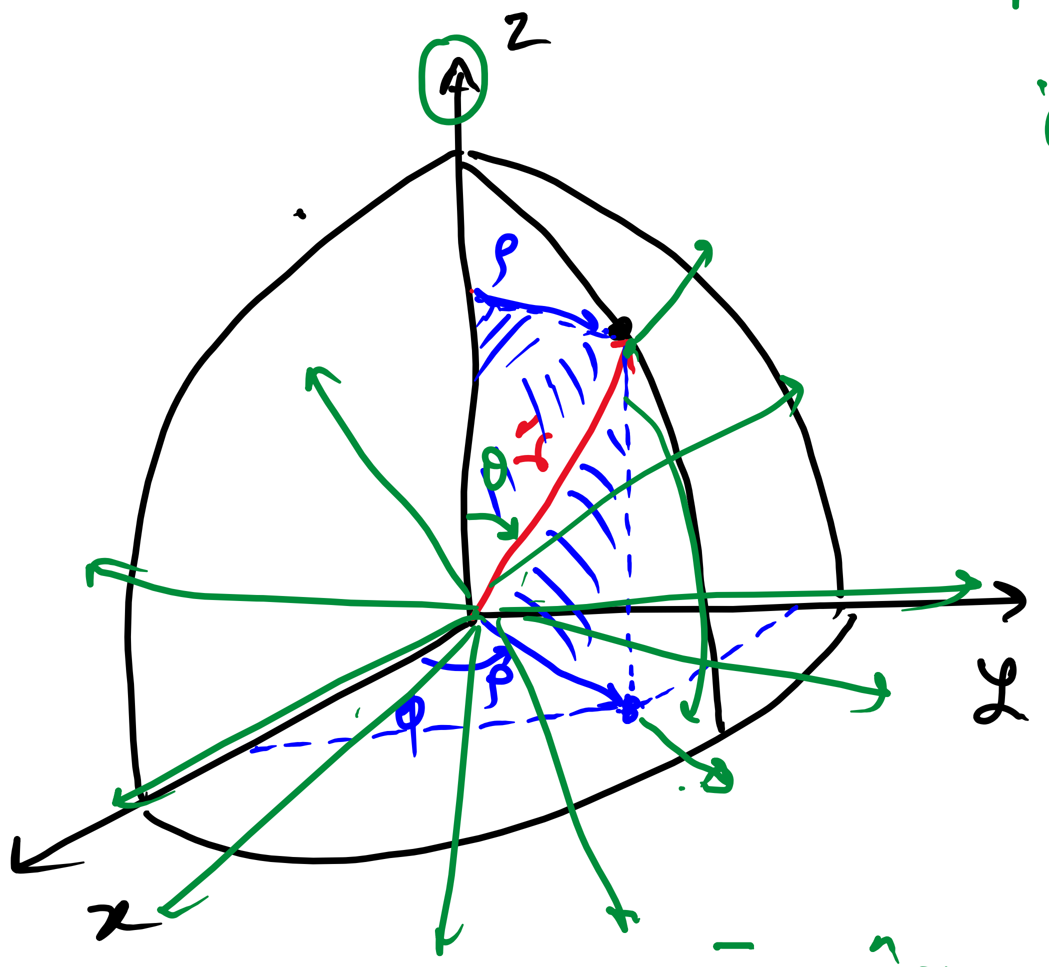
$\bar{r} \rightarrow$ Position vector
or the radius of sphere.



$(x, y, z) \rightarrow$ in Cartesian
$(r, \theta, \phi) \rightarrow$ in spherical
$(\rho, \phi, z) \rightarrow$ in cylindrical







radial vector $\bar{r}$ makes an angle 'θ' from the vertical axis.

If → $\theta = 0°$ →

$\bar{r} = \hat{z}\, z$

$\theta = 90°$ ⇒ $\bar{r} = \hat{x}\, x + \hat{y}\, y$

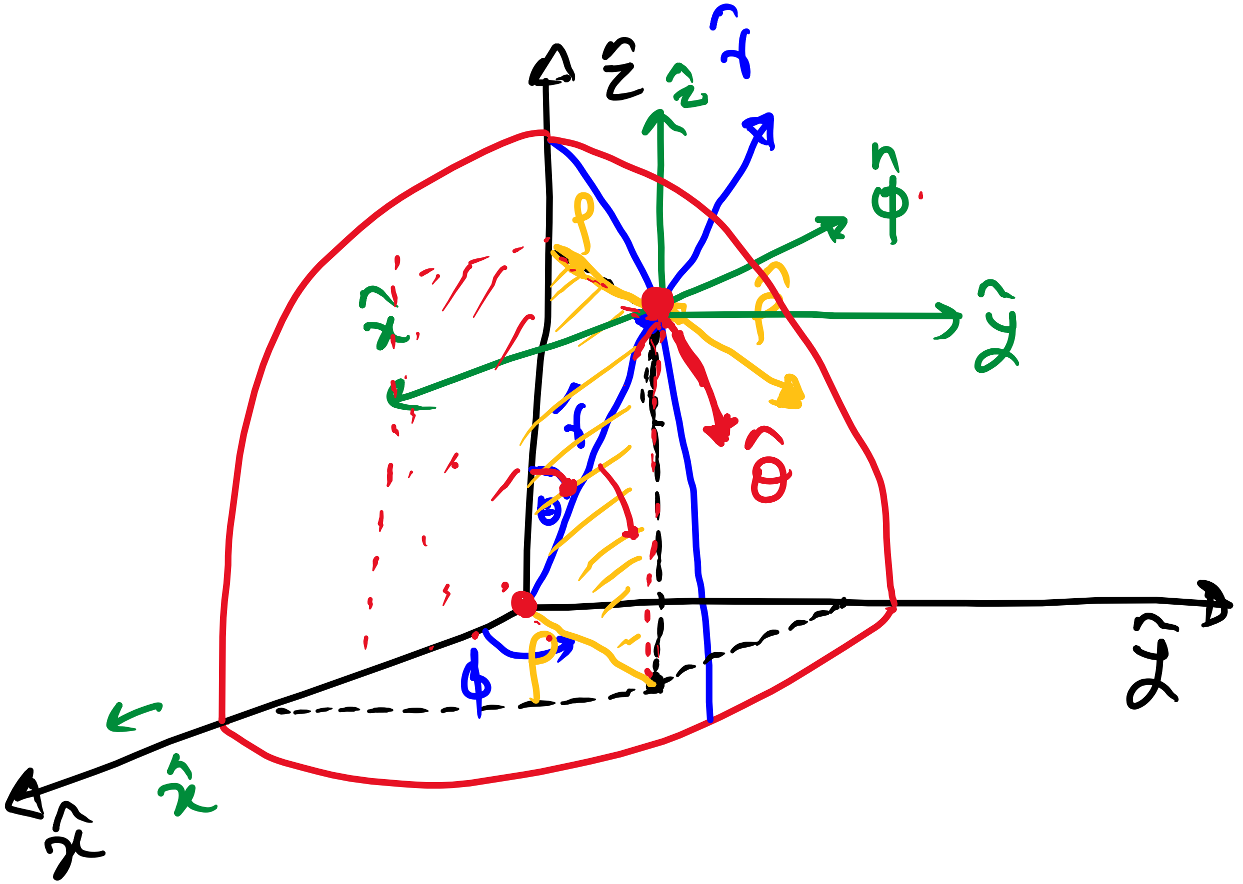
In xy-plane

$\bar{r} = \hat{x}\, x \Rightarrow \theta = 90°$

$\bar{r} = \hat{y}\, y \Rightarrow \theta = 90°$

$\bar{r} = \hat{x}\, x + \hat{y}\, y \Rightarrow \theta = 90°$

Spherical Co-ordinates - $\hat{\theta}$



$\theta = 0^\circ \Rightarrow \bar{r} \Rightarrow \bar{z}$.

$\theta = 90^\circ \Rightarrow \bar{r} \Rightarrow$ in xy-plane

$\hat{\theta} \rightarrow$ represents the increment in angle $\underline{\theta}$.

$\rightarrow \hat{x}, \hat{y}, \hat{z} \rightarrow$ rect.

$\rightarrow \hat{\rho}, \hat{\phi}, \hat{z} \rightarrow$ cyl.

$\rightarrow \hat{r}, \hat{\theta}, \hat{\phi} \rightarrow$ sphe.

$\phi = 0^\circ \Rightarrow$ Constant-ϕ Plane $\rightarrow$ xz-Plane

$\phi = 90^\circ \Rightarrow$ " " " $\Rightarrow$ yz-Plane

Spherical $\rightleftharpoons$ Cartisian.

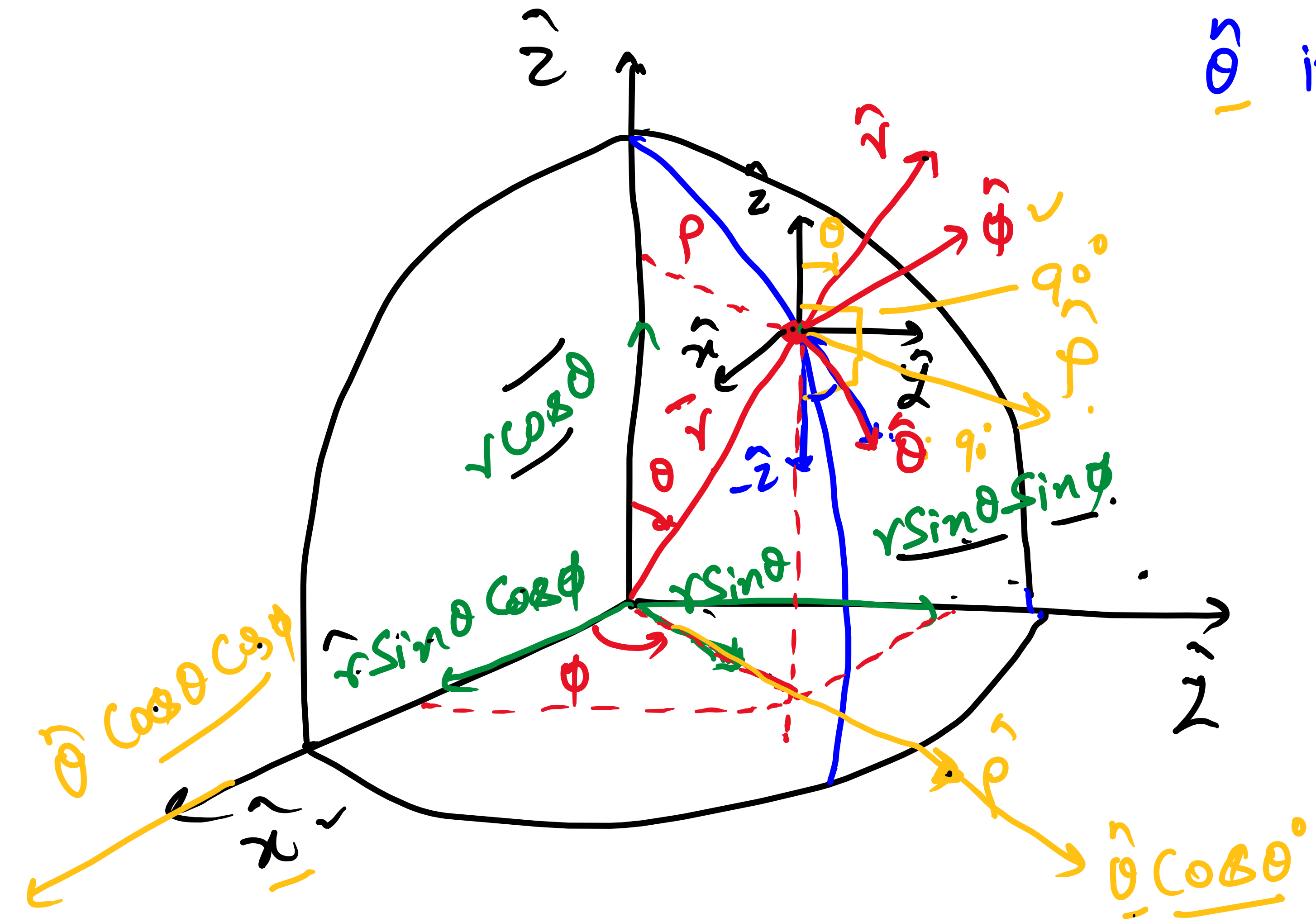
$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \hat{r}\cdot\hat{x} & \hat{r}\cdot\hat{y} & \hat{r}\cdot\hat{z} \\ \hat{\theta}\cdot\hat{x} & \hat{\theta}\cdot\hat{y} & \hat{\theta}\cdot\hat{z} \\ \hat{\phi}\cdot\hat{x} & \hat{\phi}\cdot\hat{y} & \hat{\phi}\cdot\hat{z} \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

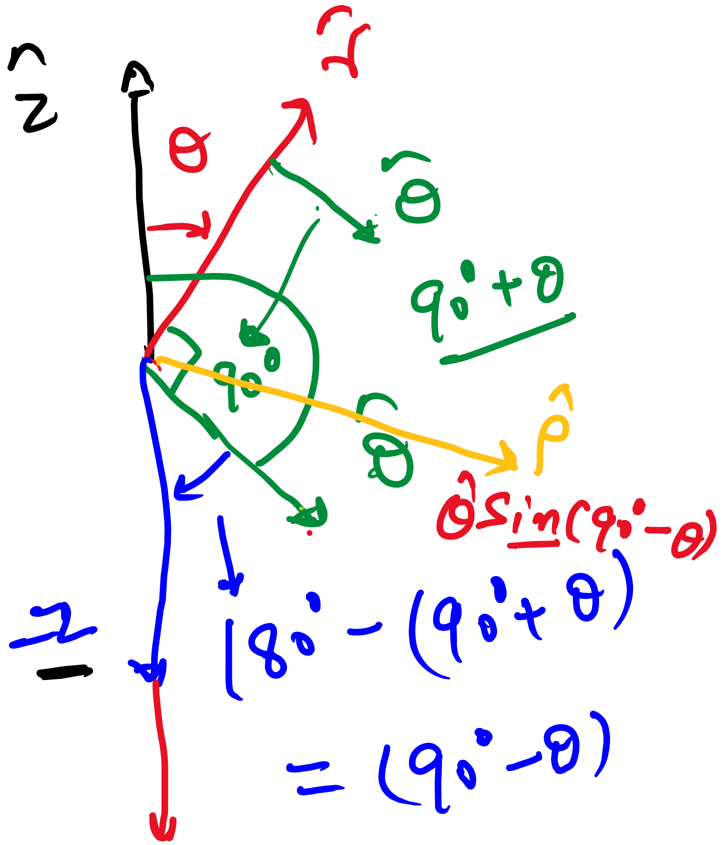
$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \hat{x}\cdot\hat{r} & \hat{x}\cdot\hat{\theta} & \hat{x}\cdot\hat{\phi} \\ \hat{y}\cdot\hat{r} & \hat{y}\cdot\hat{\theta} & \hat{y}\cdot\hat{\phi} \\ \hat{z}\cdot\hat{r} & \hat{z}\cdot\hat{\theta} & \hat{z}\cdot\hat{\phi} \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

Resolution of unit vectors

$$\hat{x} = \hat{r}\sin\theta\cos\phi + \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi$$

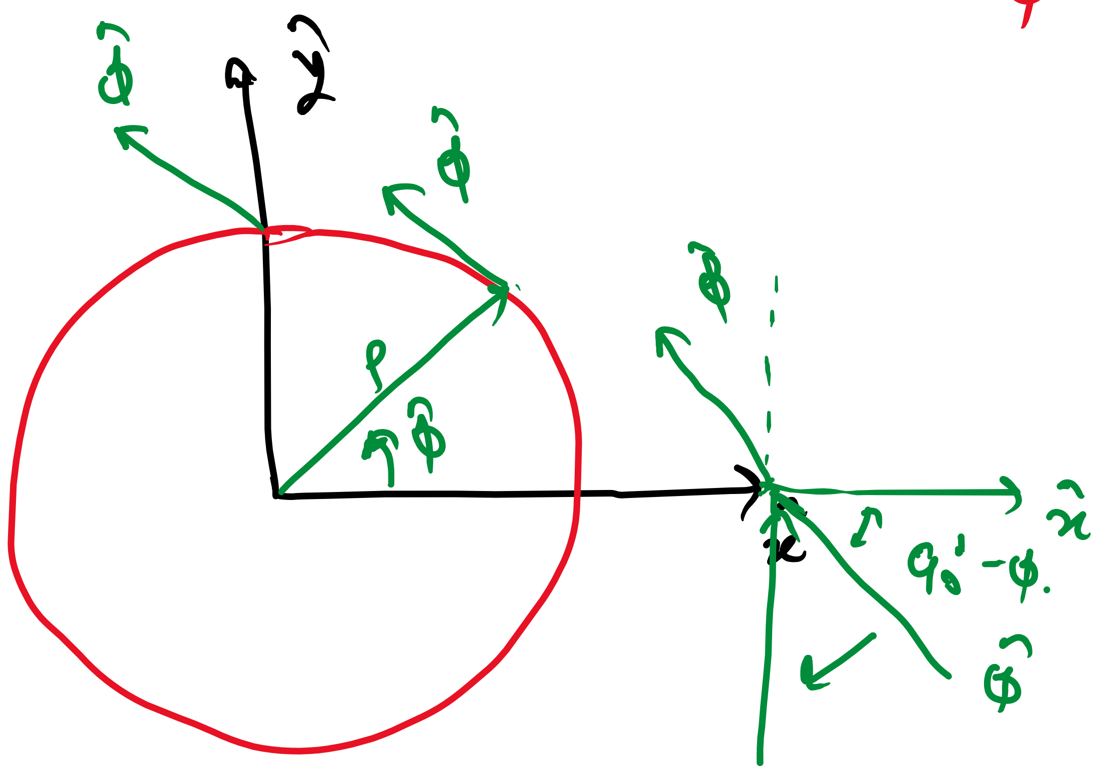
$\hat{\theta}$ in the direction of $\hat{x}$ →





$$-\hat{\theta}\cos(90^\circ-\theta) = -\hat{\theta}\sin\theta$$

$\hat{\phi},\ \hat{x}:$



$$-\hat{\phi}\cos(90^\circ-\phi)$$

$$= -\hat{\phi}\sin\phi$$

$$+\hat{\phi}\sin(90^\circ-\phi)$$

$\hat{y} = \hat{r}\sin\theta\sin\phi + \hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi.$

$\hat{z} = \hat{r}\cos\theta - \hat{\theta}\sin\theta + 0$

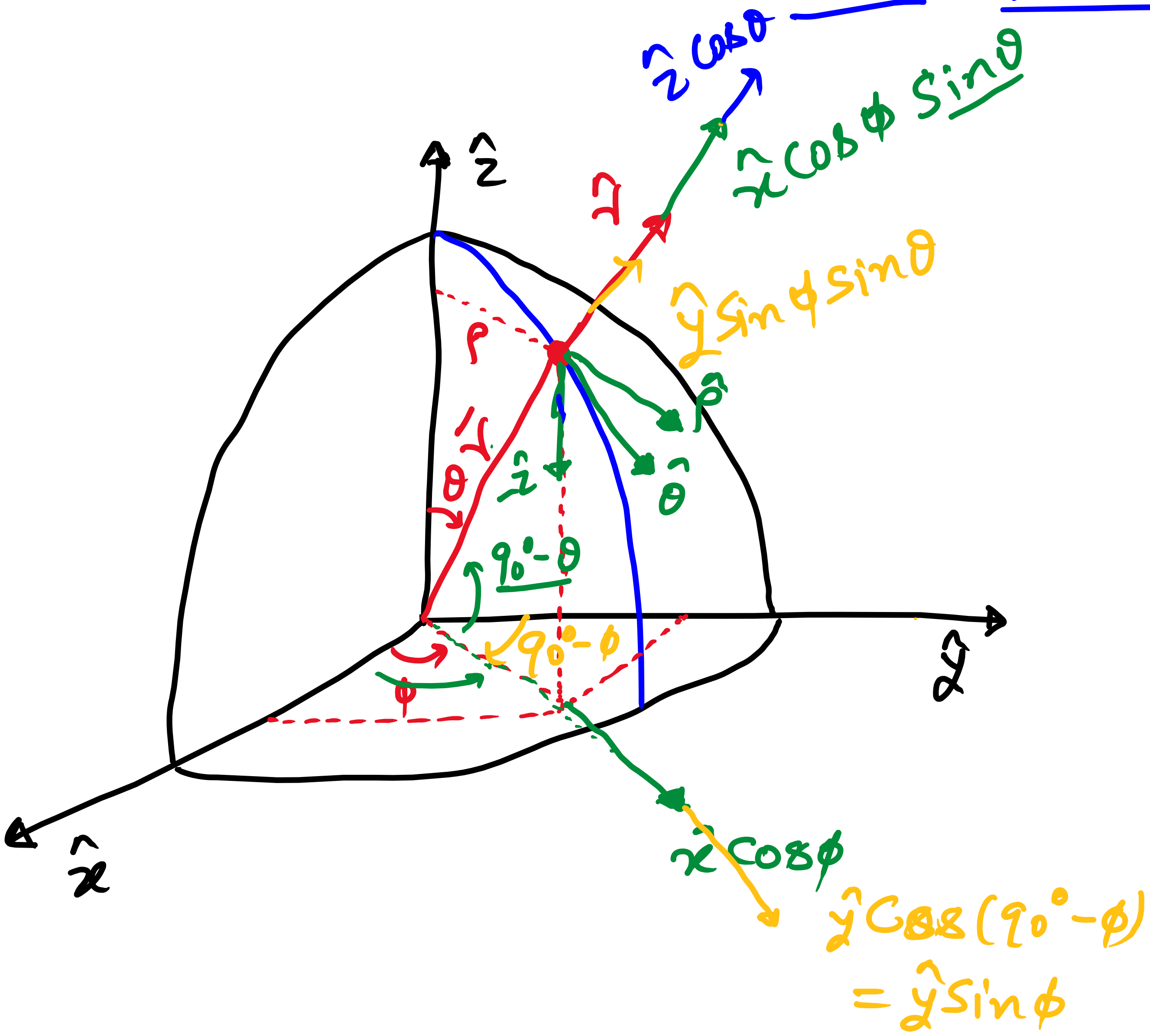
Calculate .

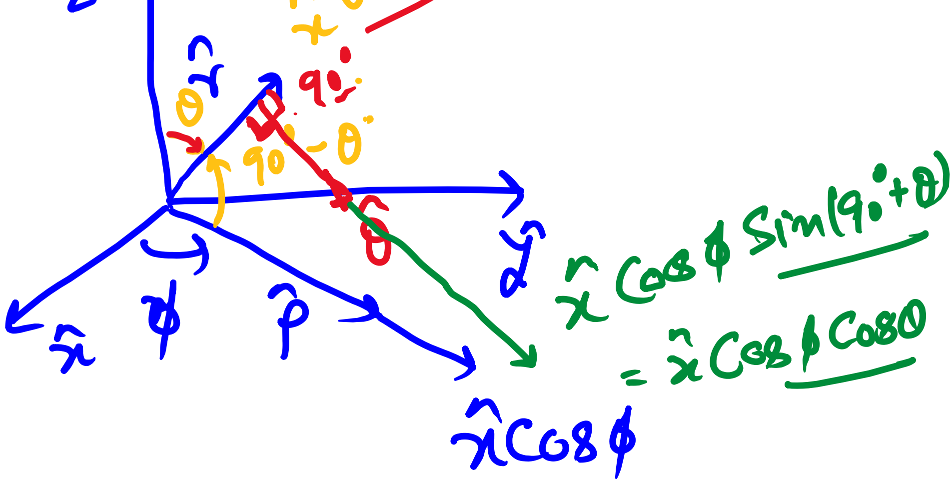
$\hat{r} = \hat{x} \qquad + \hat{y} \qquad + \hat{z}$

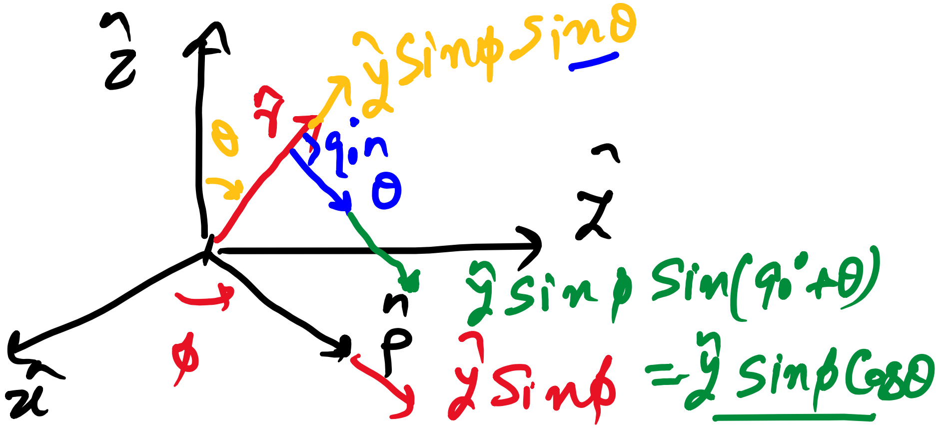
$\hat{\theta}$

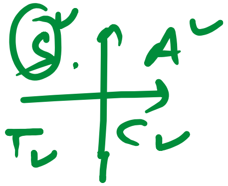
$\hat{\phi}$

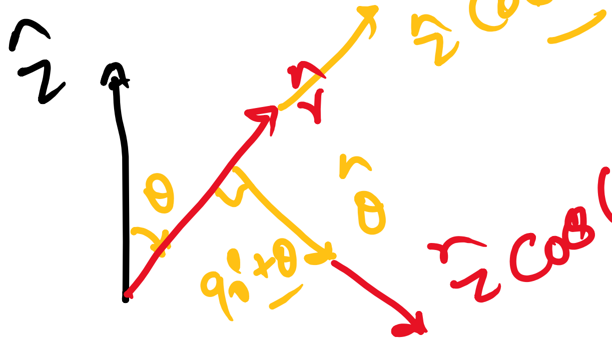
$$\hat{r} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$$



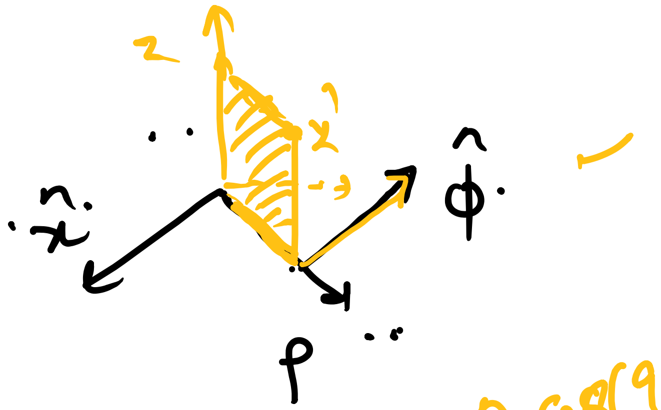




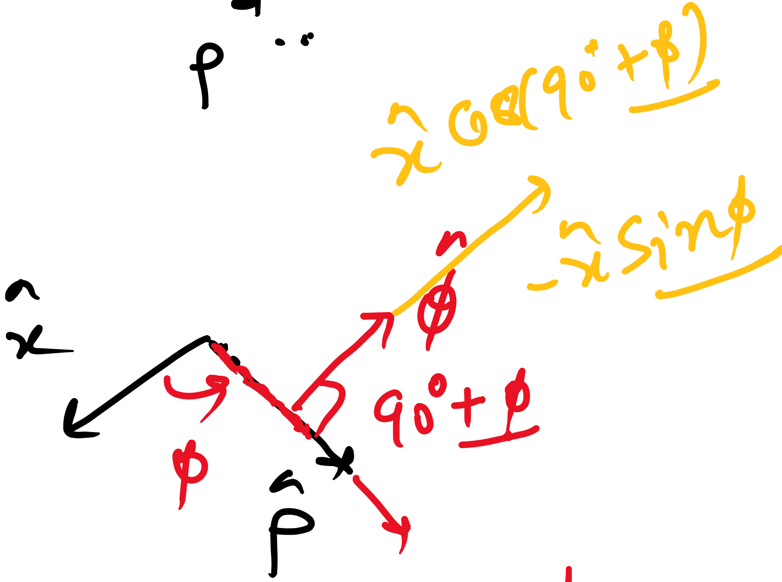




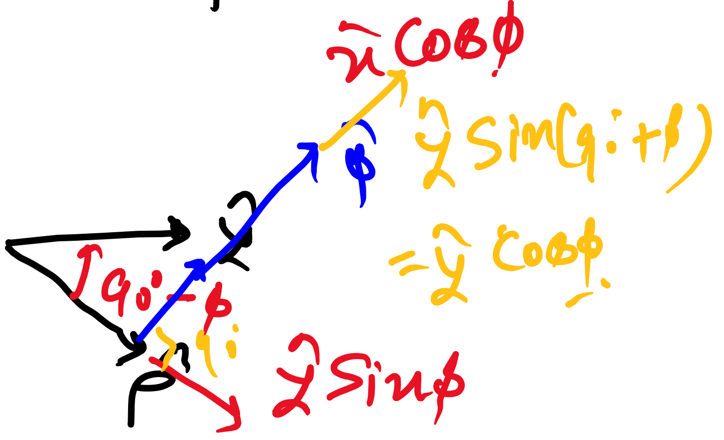
$$\hat{z}\cos(90°+\theta) = -\hat{z}\sin\theta$$



$$\hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$$



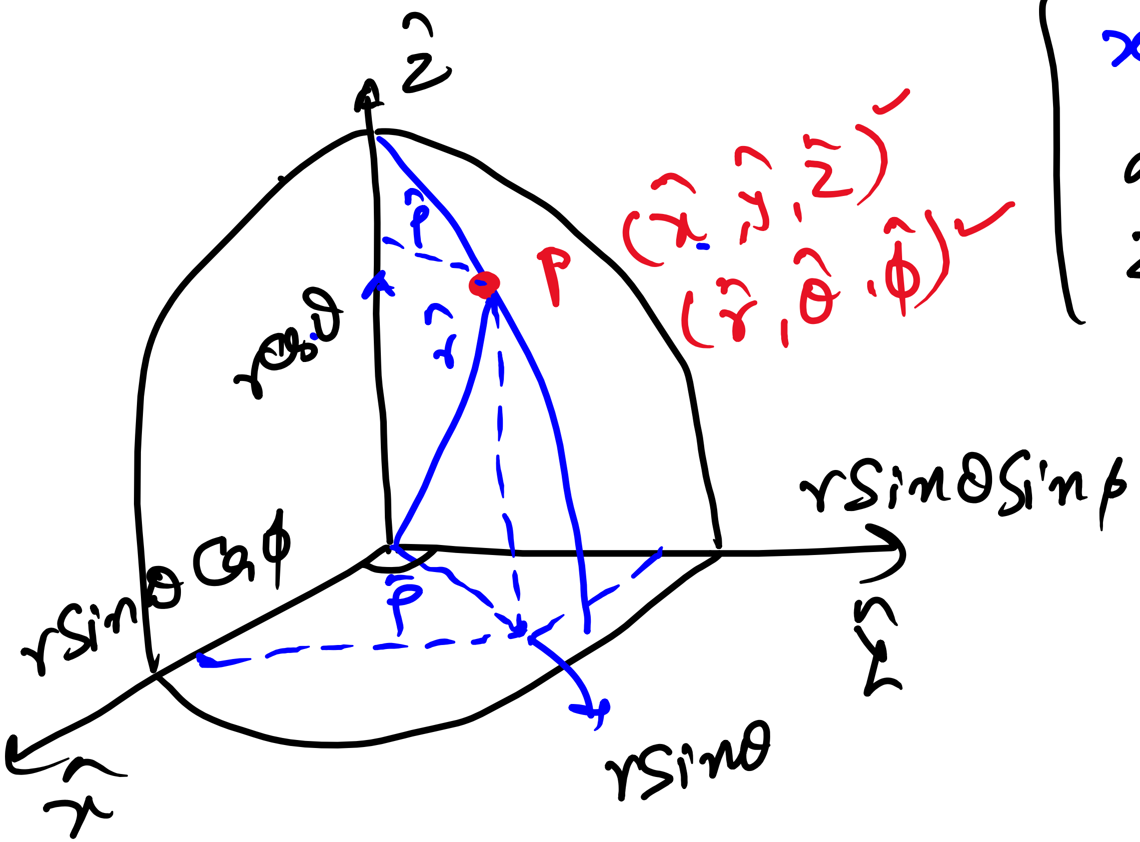
$$\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi + \hat{z}\cdot 0$$



$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \hat{x}\cdot\hat{r} & \hat{x}\cdot\hat{\theta} & \hat{x}\cdot\hat{\phi} \\ \hat{y}\cdot\hat{r} & \hat{y}\cdot\hat{\theta} & \hat{y}\cdot\hat{\phi} \\ \hat{z}\cdot\hat{r} & \hat{z}\cdot\hat{\theta} & \hat{z}\cdot\hat{\phi} \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ +\cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$



$$\begin{bmatrix} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{bmatrix}$$

$$r = \sqrt{x^2+y^2+z^2}$$

$$\theta = \cos^{-1}\left(\frac{z}{r}\right) = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

Q $P(-2, 6, 3)$ & $\vec{A} = \hat{x}\, y + (x+z)\,\hat{y}$ -
express P & A in spherical co-ordinates -

Sol →

Homework - Transform the Point & vector.
Cylindrical ⇄ Spherical

Assignment - (i) Solve all the unsolved numerical problems of ch-2 of Elements of Electromagnetics by Sadiku.
due date : - 07/Sep/2021

(ii) Solve all the solved problems of ch-2 & review questions given in the last of the chapter.